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The Estimation for Length of Closed Phase Trajectory of Two-Dimensional System of Ordinary Differential Equations

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Let us consider two-dimensional autonomous system of ordinary differential equations:

$$\dot{x} = f(x, y) \qquad \dot{y} = g(x, y) \tag{1}$$

possessing by periodic solution $\gamma : (X(t), Y(t))$ with period T, this closed curve γ being the boundary of some domain $\Omega \subset R^2$: $\gamma \equiv \partial\Omega$.

The next inequality for the length

$$L = \int_0^T \sqrt{\dot{X}^2(t) + \dot{Y}^2(t)}\, d\,t \tag{2}$$

of this curve γ proves to be true:

$$\sqrt{4\,\pi\,F(\Omega)} \leq L \leq T\,\sqrt{\overline{\dot{X}^2} + \overline{\dot{Y}^2}}\,, \tag{3}$$

where $F(\Omega) = \frac{1}{2}\oint_\gamma x\,dy - y\,dx$ is the area of the domain Ω restricted by the curve γ and $\overline{f} \equiv \frac{1}{T}\int_0^T f(t)\,d\,t$ denotes averaging over the period T of function $f(t)$. It is obvious that the left side of formula (3) is corollary of the isoperimetric inequality [1] and the right one is consequence of the Cauchy-Schwarz-Bunyakovskii inequality.

Let us now consider the system (1) with $f(x, y) = y$ and $g(x, y) = -\omega^2\,x$ under $0 < \omega \leq 1$. For this system there is the well-known periodic solution $\gamma : (\sin\omega\,t\,,\ \omega\,\cos\omega\,t)$ with period $T = \frac{2\,\pi}{\omega}$. Applying inequality (3) to it one can easily obtain that a complete elliptic integral of the second kind $E(k)$ under all $k \in [0, 1]$ obeys to the following inequality:

$$\frac{\pi}{2}\,\sqrt[4]{1-k^2} \leq E(k) \leq \frac{\pi}{2}\,\sqrt{1-\frac{k^2}{2}}\,. \tag{4}$$

The above described system is the special case of Hamilton system with Hamiltonian:

$$H(x, y) = \frac{y^2}{2} + U(x)\,. \tag{5}$$

Under wide range of potential wells $U(x)$ for systems (5) there are one-parametric families $\gamma_E\,:\,(X(t,E),\,\dot{X}(t,E))$ of periodic solutions depending on total energy E of the system, open domains Ω_E such that $\gamma_E = \partial\Omega_E$ being determined by inequality: $H(x,y) < E$. In this situation $F(\Omega_E) = 2\,\pi\,I(E)$ where $I(E)$ is the action variable of the system (5) and period of its movement is equal to $T(E) = 2\,\pi\,\frac{d\,I(E)}{d\,E}$. In this case length (2) of γ_E is some function $L(E)$ of energy E too. Moreover there are potentials $U(x)$ for which $L(E)$ can be estimated without any information about explicit temporal dependence of function $X(t,E)$. For instance under $U(x) = \frac{x^8}{8}$ for length $L(E)$ inequality (3) gives us for all $E \in [0,+\infty)$:

$$\sqrt{\frac{2\,\sqrt{\pi}}{5}\,\frac{\Gamma(1/8)}{\Gamma(5/8)}\,(8\,E)^{\frac{5}{16}}} \le L(E) \le \sqrt{\pi}\,\sqrt{\frac{\Gamma^2(1/8)}{\Gamma^2(5/8)}\,\frac{(8E)^{\frac{1}{4}}}{5} + \frac{56}{33}\,\cot\frac{\pi}{8}\,(8\,E)}$$

where $\Gamma(\ldots)$ is the Euler gamma function and

$$L(E) = 4\,(8\,E)^{\frac{1}{8}}\,\int_0^1 \sqrt{\frac{1 - z^8 + 4\,(8\,E)^{\frac{3}{4}}\,z^{14}}{1 - z^8}}\,d\,z\,.$$

The suggested inequality (3) may be useful both under solution of the second part of Hilbert's 16th problem and the weakened Hilbert's 16th problem [2, 3] and under studying of the Feynman integrals [4]. The estimation (4) can be applied to the investigation of some problems of contemporary geometric function theory (see [5] and references there).

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On Asymptotic Solution for One of the Simplest Model of Cosmological Inflation

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Let us deal with the next model of interacting gravitational field g_{ij} and scalar field φ (see [1] and references there in):

$$S[g,\varphi] = \int \left(-\frac{R}{16\,\pi\,k} + \frac{1}{2}\, g^{ij}\, \frac{\partial\varphi}{\partial x^i}\, \frac{\partial\varphi}{\partial x^j} - \frac{m^2\varphi^2}{2} \right) \sqrt{-g}\, d^4x\,. \qquad (6)$$

In functional (1) k is gravitational constant and m is mass of scalar field.

Further let us consider spatially-flat space-time with metric:

$$ds^2 = dt^2 - a^2(t)\,(dx^2 + dy^2 + dz^2) \qquad (7)$$

and spatially homogeneous scalar field $\varphi(t)$ then equations of motion for action (1) are reduced to the following system of ordinary differential equations:

$$\ddot{\varphi} + 3\,\frac{\dot{a}}{a}\,\dot{\varphi} + m^2\,\varphi = 0\,, \qquad \left(\frac{\dot{a}}{a}\right)^2 = \frac{4\,\pi\,k}{3}\,(\dot{\varphi}^2 + m^2\,\varphi^2)\,. \qquad (8)$$

Due to smallness of gravitational constant in the framework of approach developed in [2] asymptotic solution of system (3) has been constructed:

$$\varphi(t) \approx \frac{\varphi_0\,\cos m\,t}{1 + 2\,\varphi_0\,\sqrt{3\,\pi\,k}\,m\,t}\,, \qquad a(t) \approx a_0\,(1 + 2\,\varphi_0\,\sqrt{3\,\pi\,k}\,m\,t)^{\frac{\sqrt{3}}{3}}\,. \qquad (9)$$

In formulae (4) φ_0 and a_0 are initial values of scalar field and scale factor respectively of the Universe (2) .

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Moving solitons for the Lugiato-Lefever equation

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We consider travelling-wave solutions for the Lugiato-Lefever equation

$$i\psi_t + 0.5\psi_{xx} + \psi - |\psi|^2\psi = -i\gamma\psi + h$$

that are of soliton type, i.e., $\psi \to \psi_0^{\pm}$, $x \to \pm\infty$. Assuming that $\psi = u + iv$, $\xi = x - Vt$, we deal with the system

$$\begin{aligned} &0.5u_{\xi\xi} + Vv_\xi + u - u(u^2 + v^2) = \gamma v + h, \\ &0.5v_{\xi\xi} - Vu_\xi + v - v(u^2 + v^2) = -\gamma u \end{aligned}$$

Two cases are considered:

(a) If $\gamma = 0$ the system has the first integral and is Hamiltonian. This allows the existence of soliton solutions that are asymptotic to equilibrium states of saddle-focus or saddle type. In this case (i) an asymptotic expansion of soliton solution in the small-amplitude limit is given and (ii) the several branches of soliton solutions are computed and the variation of soliton shape along this branches is studied.

(b) If $\gamma \neq 0$, $V \neq 0$ and $h \neq 0$, it is argued that no soliton solution exist in generic situation. However, these solutions may exist for separate values of these parameters.

“Transparent points” in Discrete NLS Equation with Saturation

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We consider standing lattice solitons for discrete nonlinear Schrödinger equation with saturation (NLSS), where so-called transparent points were recently discovered. These transparent points are the values of the governing parameter (e.g., the lattice spacing) for which the Peierls-Nabarro barrier vanishes. In order to explain the existence of transparent points, we study a solitary wave solution in the continuous NLSS and analyse the singularities of its analytic continuation in the complex plane. The existence of a quadruplet of logarithmic singularities nearest to the real axis is proven and applied to two settings: (i) the fourth-order differential equation arising as the next-order continuum approximation of the discrete NLSS and (ii) the advance-delay version of the discrete NLSS. In the context of (i), the fourth-order differential equation generally does not have solitary wave solutions. Nevertheless, we show that solitary waves solutions exist for specific values of governing parameter that form an infinite sequence. We present an asymptotic formula for the distance between two subsequent elements of the sequence in terms of the small parameter of lattice spacing. It is in excellent agreement with our numerical data. In the context of (ii), we also derive an asymptotic formula for values of lattice spacing for which approximate standing lattice solitons can be constructed. The asymptotic formula is in excellent agreement with the numerical approximations of transparent points. However, we show that the asymptotic formulas for the cases (i) and (ii) are essentially different and that the transparent points do not generally imply existence of continuous standing lattice solitons in the advance-delay version of the discrete NLSS.

Dependency of coupling estimates obtained with partial directed coherence method on time series length

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Signal analysis is one of the most common branches of science. Much that surrounds us, all the information that we receive is presented in the form of signals. Therefore, one of the current areas of modern science is to find new methods for analyzing the measured signals. Different approaches to coupling estimation between systems of different nature were proposed in last decades, including methods for directed coupling estimation for biological data [1]. One of the most popular and perspective methods is partial directional coherence (PDC, see [2]), which was proposed in [3]. PDC was declared to show the direct influence of one time series on another, without showing indirect links. However, the method is based on construction of linear forecasting models, with unknown coefficients of those models being estimated using least squares method. Then, time-dependent coefficients are converted into frequency-dependent. Therefore its efficiency for nonlinear systems is not obvious. The goal of the current research is to determine whether this method will detect the parametric dependence of nonlinear relationships.

Here we propose a number of simple oscillating stochastic models, usually constructed by modifying the known two-dimensional and three-dimensional dynamic systems (van der Pol oscillator, Fitzhugh-Nagumo system, Morris-Lekar system, Hindmarsch-Rose system, Rosler system), connected parametrically. These models provide an opportunity to test various methods for coupling estimation, including PDC. To determine the level of significance of the results obtained, we constructed surrogates by mixing different episodes generated with different noise realizations and from various initial conditions.

The results of the performed analysis show that by means of PDC the coupling architecture can be revealed even for parametrically coupled systems. However, the method efficiency is directly proportional to time series length. With increasing length, the value of coherence increases for the links, which are actually present, and decreases for others, redundant links. At the same time, the frequency at which the peaks are observed does not change. As a result, we directly showed the PDC is a adequate tool for coupling analysis in case of nonlinearly coupled systems and formulated the criteria for its application to such signals.

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Shilnikov homoclinic loops in the Rosenzweig-MacArtur model

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In this talk we observe results obtained at the study of bifurcation set near Shilnikov homoclinic loop of a saddle-focus equilibrium in the Rosenzweig-MacArtur model describing dynamics in three-species food chain model prey-predator-superpredator. In the parameter plane of the system, the so-called *U-shaped* bifurcation curve corresponds to a homoclinic loop of a saddle-focus [1,2]. Such bifurcation curve consists of two branches the distance between which is very small (10^{-10}). These two branches are converged in the specific co-dimensional 2 point, which is called as the *periodicity hub*. The distance between these two branches is very small (10^{-10}), and thus, in two-dimensional bifurcation diagrams, it seems that the homoclinic bifurcation curve is a line starting in periodocity hub. In [3] it is shown that along homoclinic bifurcation curve there are an infinite number of periodicity hubs.

Using one-dimensional maps we explain global organizing of periodicity hubs emerging along the homoclinic bifurcation curve. We demonstrate the emergency of secondary homoclinic bifurcation curves near each such hub. Using one-dimensional maps we show how to predict the form of secondary homoclinic orbits. Finally, by explicit constructing homoclinic orbits, we confirm our prediction.

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Ghost-attractor in blinking nonlinear rotator-oscillator

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In this talk we consider the equation of blinking nonlinear rotator-oscillator

$$\ddot{\varphi} + (\lambda - a\cos\varphi)\dot{\varphi} + \sin\varphi = h(t), \tag{10}$$

where φ is the phase of oscillator, λ and a are positive parameters and $h(t)$ is a randomly switching parameter such that for i-th time interval $t \in [t_i, t_i + \tau)$ it takes values $h(t) = b$ with probability p and $h(t) = -b$ with probability $q = 1 - p$. Here b is a positive constant. This random independent switching of a parameter for small time intervals $\tau \ll 1$ has been called "blinking" by analogy with blinking eyes [1]–[4].

For $h(t) \equiv \gamma$, γ is a constant, the partition of the parameter space $D : \gamma, \lambda, a$, whose domains correspond to different partitions of the phase space, is well known [5].The values of parameter $\gamma \pm b$, which are taken by switching function $h(t)$, were chosen from the regions corresponding to the existence of globally stable rotational cycles. Since the switching period $\tau \ll 1$ is small, the system (1) can be averaged over a fast time $t' = \frac{t}{\mu}$ and the random function $h(t)$ can be replaced by its average time value $\gamma = \langle h(t') \rangle = 0$. Thus, the averaged system corresponds to the case of the system (1) with $\gamma = 0$. This averaged system has a globally stable O-cycle enclosing an equilibrium state at the origin. This cycle is absent in both autonomous systems that have rotational (in opposite directions) limit cycles.

For large switching intervals $\tau < \tau_{th}$, a globally stable attracting set appears in the blinking system (1), lying in the vicinity of the attractor of the averaged system. This attractor, which is absent in systems involved in blinking, but determining its dynamics, is a ghost attractor [6].Recall that the ghost attractor of a blinking system is an attractor (non-stationary) of a blinking system, which differs in structure or type from the attractors of the systems between which blinking occurs.

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Analysis of stochastic dynamic in autocatalytic regulation model

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We consider stochastic variant of biochemical autocatalytic reaction model [1] governed by the following system

$$\begin{aligned} \dot{x} &= v - \sigma\varphi(x,y) + \varepsilon\xi(t) \\ \dot{y} &= \alpha\varphi(x,y) - ky. \end{aligned} \tag{11}$$

The model belongs to the class of excitable and oscillatory chemical systems. In the deterministic case it possesses two dynamic regimes: quiescent and self-sustained oscillations. We examine how random noise can affect these regimes and induce stochastic phenomena. For theoretical analysis of noise-induced effects we suggest an approach combining stochastic sensitivity function technique [2] and method of confidence domain. Using this approach an analysis of excitability is carried out and supersensitive cycles in Canard-like cycles zones are found. The phenomenon of stochastic cycle splitting is also investigated by probability density distribution analyzing.

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Simplified maps modeling funnel attractors in 3D flow with one or two equlibria

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The funnel attractor in 3D systems having a saddle-focus O, whose 2D unstable manifold W^u returns in a neighbourhood $N(O)$ of this unique equlibrium point, was proposed by Shilnikov relating to his famous theorem on homoclinic orbit of O, when W^u in $N(O)$ meets 1D stable manifold W^s.

1. Consider a three dimensional system

$$\dot{v} = F(v), \quad v = (u, z), \quad u = (x, y), \tag{12}$$

having a unique eqilibrium point $O(v = 0)$. Let the Jacobian $F_v(O)$ be a diagonal matrix $A = \begin{pmatrix} -\lambda & -\omega & 0 \\ \omega & -\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\lambda > 0, \omega > 0$. Hence, the saddle-focus has the 2D stable manifold W^s which is tangent to $\{z = 0\}$ at O and the 1D unstable manifold W^u tangent to $\{u = 0\}$ at O.

Consider the natural norm $\rho = \sqrt{x^2 + y^2}$ and a neighbourhood $N(O) = \{0 \leq z \leq 1, \rho \leq 1\}$. We prove the next assertion.

Theorem 1. *Assume that each trajectory of the system (1) with initial condition at the cylinder $C = \{\rho = 1, 0 \leq z \leq 1\}$ returns into the cross-section $D = \{z = 1, \rho \leq 1\}$ after a finite transition time. Then the flow of the system (1) generates a map $F : D \to D$*

being a composition $F = hf$, where h is an arbitrary diffeomorphism and $\bar{u} = f(u)$ has the explicit form

$$\begin{aligned} \bar{x} &= (a_0 + a_1\rho^\alpha)\cos(\varphi - \omega \ln \rho), \\ \bar{y} &= (a_0 + a_1\rho^\alpha)\sin(\varphi - \omega \ln \rho), \end{aligned} \tag{13}$$

where $\varphi = \arctan \dfrac{y}{x} + \dfrac{\pi}{2}(1 - signx)$, a_0, a_1, $\alpha = \dfrac{1}{\lambda}$ are parameters, $\alpha > 2$ is Shilnikov condition.

In the trivial case $h = id$ the map (2) reduces to the map $\bar{\rho} = \rho + q\rho^\alpha$ which stable fixed point $\rho = \rho^*$ defines the stable invariant circle with the turn map in it

$$\bar{\varphi} = \varphi - \omega \ln \rho^*. \tag{14}$$

Breakdown of the system (1) torus corresponding to the circle $\rho = \rho^*$ occurs even at trivial shift $h : (x, y) \to (x + \varepsilon, y)$ before the Shilnikov homoclinic orbit bifurcation for $\varepsilon = a_0$.

2. Let the system (1) has two equilibria: one is the saddle-focus O in the origin with $F_v(O) = A$ and another is the saddle $E(v = v^*)$ such that the Jacobian $F_v(v^*) = B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\nu \end{pmatrix}$, and the saddle E has 1D unstable manifold $\mathcal{W}^u$ and 2D stable manifold $\mathcal{W}^s$. Assume that there exists a heteroclinic curve $l_{he} = W^u \cap \mathcal{W}^s$, and both branches of the unstable manifold $\mathcal{W}^u$ hit the cross-section disk D of the saddle-focus. In the case $\nu < 1 < \beta$ the saddle E having "homoclinic butterfly" has the Lorenz-type attractor with the model map of its local cross-section $S = (\xi, \eta) \in \mathbb{R}^2$ can be written in the form

$$\begin{aligned} \bar{\xi} &= (\gamma + \delta|\xi|^\nu)\mathrm{sign}\xi, \\ \bar{\eta} &= q_0\mathrm{sign}\xi + q_1|\xi|^\beta\eta. \end{aligned} \tag{15}$$

Combing the properties of the flow and of the local behavior of the stable and unstable manifolds from (2)-(4) we derive a heuristic map of the disk D into itself

$$\begin{aligned} \bar{x} &= a(1 - (1 - x)^\nu) \quad \text{for} \quad x \in [0, 1), \\ \bar{x} &= a(-1 + (1 + x)^\nu) \quad \text{for} \quad x \in [-1, 0) \\ \bar{\rho} &= k\varphi + \rho_0 + m|\varphi - \pi|\rho^\alpha, \quad \varphi = \pi x \in \mathbb{S}^1, \end{aligned} \tag{16}$$

where k, ρ_0, m, a are positive parameters.

Let $k_1 = -k\pi + \rho_0 > 0$ and 1D map $\bar{\rho} = k_1 + m\pi\rho^{\alpha}$ for $\alpha > 2$ has the stable fixed point ρ_1. Then the map (5) is the map of the annulus $\mathcal{A} = \{k_1 < \rho < \rho_1\}$.

Theorem 2. *For $a > 2$, $\alpha > 2$ the image $F\mathcal{A} \subset \mathcal{A}$ is a disk containing singularly hyperbolic attractor of Ressler-type.*

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Symmetry broken states in an ensemble of coupled pendulums

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We consider the rotational dynamics in an ensemble of identical globally coupled pendulums [1]. This model is essentially a generalization of the standard Kuramoto model, which takes into account the inertia and the intrinsic nonlinearity of the community elements [2, 3]. There exist the wide variety of in-phase and out-of-phase regimes. Many of these states appear due to instability of in-phase rotational mode that leads to partially broken symmetry in the considered model. Our theoretical analysis allows to find the boundaries of the instability domain for ensembles with arbitrary number of pendulums. In the case of three elements, for system control parameter sets corresponding to the unstable in-phase rotations we numerically find a number of out-of-phase rotation modes and study in detail their stability and bifurcations. We demonstrate that chaotic behavior arises throw the cascade period doubling bifurcations and torus destruction. As a result, we obtain a sufficiently detailed picture of the symmetry breaking and existence of various regular and chaotic states in an ensemble of identical globally coupled pendulums. In particular, we shows that the family of symmetry-broken states includes chimera attractors. In these regimes, one part of the oscillators moving irregularly is synchronized, and the other pendulums rotate separately [4]. We directly demonstrate example of chimera states in the system under discussion for the cases of three and four number of elements.

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Exponentially small splitting of separatrices associated to 3D whiskered tori with cubic frequencies

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The splitting of invariant manifolds of whiskered (hyperbolic) tori with three frequencies in a nearly-integrable Hamiltonian system, whose hyperbolic part is given by a pendulum, is studied. We consider a 3-dimensional torus with a fast frequency vector $\omega/\sqrt{\epsilon}$, with $\omega = (1, \Omega, \Omega^2)$ where Ω is a cubic irrational number whose two conjugates are complex (for instance, the real root of $z^3 + z - 1$). Applying the Poincaré-Melnikov method, we carry out a careful study of the dominant harmonics of the Melnikov function. This allows us to provide an asymptotic estimate for the maximal splitting distance, which is exponentially small in ε, and valid for all sufficiently small values of ϵ. The function in the exponent turns out to be quasiperiodic with respect to $\log \epsilon$, and can be explicitly constructed from the resonance properties the frequency vector ω. In this way, we emphasize the strong dependence of our results on the arithmetic properties of the frequencies.

Topological states of matter as bifurcations of trajectories of Schrödinger equation in periodic potential

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Topological physics has been one of the most rapidly developing fields in the last decade. The main idea is to study connections of the vector bundle generated by a gauge field for the trajectories of a dynamical system determined by Schrödinger equation:

$$i\hbar \frac{d\psi(x)}{dt} = H\psi(x)$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + U,\ U\psi(x+R) = U\psi(x)$$

The periodicity of the potential U demands the solution to have a form, $e^{i\pi k u_k(x)}$. Hence the spectrum of the Hamiltonian is also parametrised by k, which can be associated with the gauge field. For a fermionic spinless system, multiple (degenerate) eigenvalues are not allowed, but in numerical study such a situation occurs and can be protected by crystal symmetries which are used in the simulation of the periodic potential. This problem is resolved by introducing a spin degree of freedom. In this case electrons in a crystal show a spin dependent behavior such as spin textures, the spin Hall effect, and spin ordering (magnetism). The development of a predictive analysis of such properties in real compounds is one of the most important goals for theoretical physics. Here we introduce mathematical tools that physicists use for attacking these issues; emphasizing the concept of Berry curvature on the Hamiltonian spectrum and the Kubo computational formalism for the calculation of certain transport properties (and its connection with degenerate points in spectrum). The spin dependent properties mentioned above are essentially a bifurcation of the quantum system from the non-ergodic to ergodic regime and thus can be studied in the dynamical system theory environment in combination with topological tools used now and we aim to discuss the open questions of this field.

Attractors and repellers in a system of two nonidentical phase oscillators with adaptive couplings

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We present the results of studying the attractors and repellers in a system of two nonidentical phase oscillators with adaptive couplings. It is shown that, depending on the parameters, the system demonstrates regular, chaotic, and probably mixed dynamics. It was found that the parameter of detuning of the oscillators's eigenfrequencies, which increases the "degree of instability" of the system, is a crucial factor for the existence of mixed dynamics in this system. It is shown that the phenomenon of mixed dynamics in this system is structurally stable.

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Multistability and Hyperchaos in the Dynamics of Two Coupled Contrast Agents

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In this talk we consider a model of two spherical gas bubbles interacting with each other via the Bjerknes force. We suppose that the bubbles are encapsulated into a visco-elastic shell and we take into account liquid's compressibility and viscosity [1,2]. We also assume that the bubbles are driven by external periodic pressure field. In such setting this model corresponds to the dynamics of ultrasound contrast agents. Though the dynamics of one oscillating bubble has been studied (see e.g. [3-5]), the dynamics of two coupled bubbles has not been thoroughly considered. Therefore, the main goal of this talk is to study typical bifurcation scenarios in this model of two coupled bubbles and also show that their dynamics can be multistable.

First of all, we show that the dynamics of two coupled bubbles can be of four types. Namely, they can exhibit periodic, quasi-periodic, chaotic and hyperchaotic oscillations. It is interesting that some of these attractors can mutually co-exist, which means that the dynamics of bubbles is multistable. For example, we find co-existence of a hyper-chaotic and chaotic attractors. Thus, there is hidden hyperchaos in the considered model. Then, we compute two-parametric charts of the Lyapunov exponents for the values of initial conditions obtained with the help of the numerical continuation method. These charts shows the dependence of the dynamics' type on the distance between bubbles and the magnitude of the external pressure field and help us to reveal a new bifurcation scenario leading to the appearance of hyperchaotic oscillations of bubbles.

We show that the transition from periodic to hyperchaotic behavior in the model is associated with the following bifurcations scenario. A stable periodic orbit undergoes Neimark-Sacker bifurcation and the stable quasiperiodic regime (torus) appears. Then, in the accordance with one of Afraimovich-Shilnikov scenario [6], the stable torus gives rise to the torus-chaos attractor with one positive Lyapunov exponent. The next step in the scenario is the emergency of the so-called stability window inside the torus-chaos attractor – when a stable periodic orbit becomes an attractor of the system. Further, this stable orbit undergoes secondary Neimark-Sucker bifurcations after which a stable torus appears again and the periodic orbit becomes of a saddle-focus type with three-dimensional unstable manifold. Finally, the stable torus is destroyed (again with the accordance to Afraimovich-Shilnikov scenario) and homoclinic Shilnikov attractor [7], containing periodic saddle-focus orbit with its unstable manifold appears. In the neighborhood of such saddle-focus volumes are expanded, and thus, passing near the saddle-focus orbit, trajectories have two positive Lyapunov exponents. Thus, we can conclude that the emergency of Shilnikov homoclinic attractors explains the appearance of hyperchaotic behavior in the system. We suppose that such bifurcation scenario is also typical for other models possessing hyperchaotic behavior.

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On the approximation of cylinder map by non-autonomous Hamiltonian system

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We consider approximation of the cylinder map

$$\begin{aligned}\bar{\theta} &= \theta + \alpha\bar{x} + \beta\bar{x}^2 - \bar{x}^3 \\ \bar{x} &= x + K\sin\theta\end{aligned} \tag{17}$$

non-autonomous Hamiltonian system with Hamilton function

$$H = \alpha x^2/2 + \beta x^3/3 - x^4/4 + K\cos\theta + \varepsilon x \sin\nu t \tag{18}$$

where α, β, K, ν are parametres and ε is small parameter. Using the program WInSet [1], we compare behaviour of invariant curves of the cylinder map (17) and Poincare's map for Hamiltonian systems with Hamilton function (18).

In the Hamiltonian system when $\varepsilon \neq 0$ the separatrices of periodic points transversely intersect as well as for the cylinder map. This leads to complex dynamics in the neighborhood of the separatrices. For approximation with $\varepsilon = 0$ the separatrices coincide.

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On the domain of existence of Lorenz-like attractors in a nonholonomic model of Celtic stone

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One of the most important problem in the theory of dynamical systems is the study of chaotic dynamics, and, specially, strange attractors. Such chaotic sets are found in many systems, including systems from applications. In this talk, we consider the nonholonomic model of the Celtic stone possessing strange attractors of Lorenz type. As we know, this model is the first model from applications in which such attractors were found [1,2].

We find the domain of existence of discrete Lorenz attractors in a nonholonomic model of Celtic stone. In addition, new examples of Lorenz-like attractors in this model, such as the discrete Lorenz attractor with lacunae and the discrete Lorenz attractor "without holes" are presented. We also study bifurcations leading to the birth and break-down of discrete Lorenz-like attractors.

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On bifurcations of two-dimensional diffeomorphisms with a quadratic homoclinic tangency to a nonhyperbolic saddle

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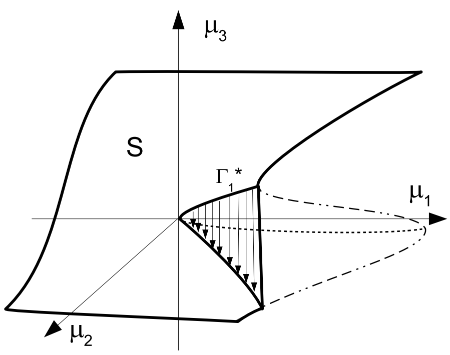
We consider a C^r-diffeomorphism f_0 ($r \geq 4$) satisfying the following conditions:

A) f_0 has a fixed point O that is a nonhyperbolic saddle with multipliers $\lambda_1 = \lambda$, where $0 < |\lambda| < 1$, $\lambda_2 = 1$, and with the second Lyapunov value $l_3 > 0$.

B) Invariant manifolds $W^u(O)$ and $W^s(O)$ of O have a (single-round) quadratic tangency at the points of a homoclinic orbit Γ_0.

Conditions A)–B) define a codimension 3 locally connected bifurcation surface in the space of two-dimensional diffeomorphisms, and, hence, for studying bifurcations of f_0 we must consider 3-parameter families. Let f_μ, where $\mu = (\mu_1, \mu_2, \mu_3)$, be such a family which unfolds generally degenerations given by conditions A)–B).



Theorem 1. Let U be a a sufficiently small neighborhood of the origin in the (μ_1, μ_2, μ_3)-parameter space. Then in U there is a two-dimensional discontinuous surface S (with the edge on the curve $\Gamma_1^* \,:\, \mu_3 = (\mu_1/2)^{1/3}, \mu_2 = -3(\mu_1/2)^{2/3}$) such that f_μ has a (single-round) quadratic homoclinic tangency either to a hyperbolic saddle fixed point or to a saddle-node fixed point when $\mu \in \Gamma_1^*$.

Theorem 2. In the (μ_1, μ_2, μ_3)-parameter space, in any sufficiently small neighborhood $U(\mu = 0)$ there is infinitely many nonintersecting domains Δ_k such that at $\mu \in \Delta_k$ the diffeomorphism f_μ has asymptotically stable single-round periodic orbit.

1) Boundaries of Δ_k correspond to codimension 1 bifurcations for single-round periodic orbits – saddle-node and period doubling ones.

2) Domains Δ_k accumulate to the surface S as $k \to +\infty$.

On discrete Shilnikov attractors in a system of symmetrically coupled oscillators

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Systems of globally coupled phase oscillators can demonstrate various type of complex behavior. If oscillators and interaction between them are identical, a coupling function is a primary source of complexity. It was shown that chaotic attractors emerge when a coupling function is non-Kuramoto: a biharmonic coupling (also called Kuramoto-Daido or Hansel-Mato-Meuner)leads to chaos in the case of five oscillators [1]. For this system, we show that the transition to chaos follows the scenario of the creation of a discete Shilnikov attractor [2]— a homoclinic attractor based on a saddle-focus fixed point with a two-dimensional unstable manifold.

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On computationally effective methods for verification of the attractor pseudohyperbolicity

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In this talk, we propose two effective methods of verification of the attractor pseudohyperbolicity [1, 2, 3]. Namely, it helps us to check the continuity of the subspaces of strong contraction E^{ss} and volume expansion E^{cu} in time complexity less than the complexity $O(d \cdot n^2)$ of the simplest approach, where n is the number of attractor points in R^d. The first way is based on the $k-d$-tree data structure, which helps to reduce the complexity to $O(d^2 \cdot n^{2-\frac{1}{d}})$. The second one uses linear-logarithmic sorting algorithms, which gives the complexity $O(d \cdot W_\epsilon \cdot n \cdot log(n))$, where $W_\epsilon << n$ is the maximum number of attractor points, located in ϵ-strip. We suppose to use the proposed algorithms for speeding up computations for two-parametric pseudohyperbolic diagrams, which gives ideas of computer-assisted proof for attractor pseudohyperbolicity.

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On discrete Lorenz and Shilnikov attractors in the modified oscillator of Anishchenko-Astakhov

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In this work we show that the transition to chaos in the modified four-dimensional oscillator of Anishchenko-Astakhov [1] can be associated with the appearance of homoclinic discrete attractors of Shilnikov and Lorenz types. Discrete Shilnikov attractors in this model appear in the accordance with the scenario proposed in [2]. For the discrete Lorenz attractor we proposed a new scenario. The main stage of this scenario is the merger of two components of Shilnikov attractor of period two with the homoclinic structure of the saddle fixed point of a type (2,1) i.e. with two-dimensional stable and one-dimensional unstable invariant manifolds. We show that discrete Lorenz attactor in the model appears in the accordance with the proposed scenario.

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On quasi-periodic perturbations of pendulum-type equation

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Quasi-periodic two-frequency perturbations of a pendulum-type equation close to an integrable one are considered. The behavior of the solutions of this equation in the neighborhood of the resonance levels is studied. The conditions for the existence of resonance quasi-periodic solutions (two-dimensional resonance tori) are found.

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Stability and bifurcations of solutions for the nonlocal erosion equation in the case of a small spatial deviation

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Here nonlocal equation erosion (see. [1]) as a

$$\begin{gathered} u_t = au_{xx} - cw_x + b(u-w) + b_1(u-w)w_x + b_2(w_x)^2 + \\ + b_3(u-w)(w_x)^2 - b_4\left(w_x\right)^3, \end{gathered} \tag{19}$$

where $u = u(t,x)$ - normalized deviation from the flat front of the target, $w = u(t, x - h)$, $h \in R$. The coefficients a, c, b_1, b_2, b_3 characterize the conditions under which the target is processed. It is considered that $a, c > 0$. The spatial deviation h was considered small, i.e. $0 < h \ll 1$. The equation (19) was considered together with periodic boundary conditions

$$u(t, x + 2\pi) = u(t, x). \tag{20}$$

If we supplement the boundary problem (19), (20) with initial conditions

$$u(0, x) = f(x), \tag{21}$$

where $f(x) \in H_2^2$, then from the results of [2] it follows that the mixed problem (19), (20), (21) is locally correctly solvable. This means that space H_2^2 can be chosen as the phase space of solutions of the boundary value problem (19), (20).

In [3] for the boundary value problem (19), (20) following questions were investigated: the stability of homogeneous equilibrium states, of possible local bifurcations in the neighborhoods of homogeneous equilibrium states, asymptotic formulas for solutions.

Such methods as: the method of integral manifolds, Poincare–Dulac normal forms, and asymptotic methods of analysis were used to solve the arising bifurcation problems and to study dynamical systems with infinite-dimensional phase space (the space of initial conditions).

The results of this work confirm that the mechanism of formation of a non-uniform relief is a mechanism of self-organization and the reason for the appearance of a non-uniform relief is that homogeneous equilibrium states lose their stability.

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On numerical methods of pseudehyperbolicity check

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In this work a new light method of pseudohyperbolicity check (LMP-method) is presented. The method checks the necessary conditions of pseudohyperbolicity based on the quite simple procedure of computing of Lyapunov exponents. The ouput of the method is an LMP-graph that is used to make decision whether a particular attractor possess a pseudohyperlic structure or not. Using this method, it was possible to check the properties of many well-known attractors such as the Henon attracor, Lorenz attractor, Chen attractor, etc.

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Nontrivial invariant manifolds of the Kuramoto-Sivashinsky equation and the Galerkin method

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Consider the boundary value problem (see [1,2]) for one of the original versions of the Kuramoto-Sivashinsky equation

$$u_t + u_{xxxx} + au_{xx} + 2uu_x = 0,\ a \in R, \tag{1}$$

$$u(t,0) = u(t,\pi) = u_{xx}(t,0) = u_{xx}(t,\pi) = 0. \tag{2}$$

Let $a = m^2 + \dfrac{\varepsilon}{12m^2}$, where ε – a small positive parameter, $m = 1, 2, 3, \dots$.

The validity of the statement follows from the works [3,4].

Theorem. There exists a positive $\varepsilon = \varepsilon(m)$ such that for all $\varepsilon \in (0, \varepsilon_m)$ the boundary value problem (1), (2) has two equilibrium states

$$u_\pm(x,\varepsilon) = \pm m\varepsilon^{1/2}\sin mx - \frac{\varepsilon}{12m}\sin 2mx \pm \frac{1}{288m^3}\varepsilon^{3/2}\sin 3mx + O(\varepsilon^2).$$

These both equilibrium states are asymptotically stable at $m = 1$ and saddle if $m \neq 1$.

It should be reminded that in the works [1,2] the question of the existence and properties of finite-dimensional global attractors was studied. These results became the basis for the wide use of the Galerkin method in analyzing the dynamics of solutions of the boundary value problem (1), (2).

We assume that

$$u(t,x) \approx \sum_{k=1}^{m} u_k(t) \sin kx. \tag{3}$$

Applying the standard procedure of the Galerkin method, we obtain a system of m ordinary differential equations

$$\dot{u}_k = -k^4 + (m^2 + \frac{\varepsilon}{12m^2})k^2 + k \sum_{p-q=k} u_p u_q - \frac{k}{2} \sum_{p+q=k} u_p u_q, \tag{4}$$

where $k, q, p = 1, 2, \ldots, m$. It is easy to verify that system (4) has a solution $u_1 = \ldots u_{m-1} = 0$, where $u_m(t) = \alpha \exp(\frac{\varepsilon}{12}t), \alpha \in R$, and the corresponding approximate solution (3) takes the following form $u_m(t,x) = \alpha \exp(\frac{\varepsilon}{12}t) \sin mx$. Wherein

$$\lim_{t\to\infty} \int_0^{\pi} u_m^2(t,x)dx \to \infty.$$

Moreover, the system (4) does not have in a sufficiently small neighborhood of the zero equilibrium point nontrivial equilibrium points.

The last remarks show that the dynamics of the solutions of the system (4) do not fully correspond to the dynamics of solutions of the boundary value problem (1), (2).

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On skew products over a quasi-periodic shift of the circle

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As is known, skew products over irrational shift on the circle arise as Poincaré mappings for quasi-periodic nonautonomous systems with two incommensurable frequencies. Many results for such skew products exists when the related bundle over circle is linear (reducibility, localization, spectral properties, etc.). Much work was done for the structure of 1-dimensional skew products (with 1-dimensional leaves): non-smooth bifurcations, rotation sets, and so forth. In the talk I intend to touch the topic of structural dynamics for such skew products, including classification problems.

The first topic is the dynamics of skew products with the circle as a leaf. Here there is a simple way to distinguish vertically hyperbolic diffeomorpisms. For such diffeomorphisms the classifying theorem is valid: the rotation number for a shift, the integer and some permutation are the complete set of invariants. Examples are presented when absence of vertical hyperbolicity leads to a dimension inhomogeneity of the invariant sets. Analogs of such skew products in greater dimensions will be discussed (analogs of gradient-like skew products, Morse-Smale products).

The situation is much more involved when the leaves are n-dimensional submanifolds

$(n \geq 2)$ and intersection of non-wandering set with leaves are infinite. Then even in the case of vertical hyperbolicity the dynamics can be very complicated. Related examples will be discussed.

Canard solutions in a one-dimensional array of FitzHugh-Nagumo neurons

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We consider an array of three FitzHugh-Nagumo neurons coupled unidirectionally by chemical synapses. The first neuron of the array is at rest. Under its influence the second neuron can be excited and depending on the parameters yields either small quasiharmonic or canard or large anharmonic oscillations. The canard solutions exist in a narrow region on control parameters plane near the Andronov-Hopf bifurcation curve, and, most interestingly, spend long times near repelling slow manifolds. The behavior of the third neuron depends on the type of oscillations of the second neuron. It has been shown that an invariant torus or torus canards can exist in the phase space of the third neuron.

Diffeomorphisms preserving Morse-Bott foliations

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Let M be a smooth compact manifold and $\mathcal{F}$ be a codimension one foliation on M having singular leaves of Morse-Bott type. Denote by Σ the set of singular leaves of $\mathcal{F}$. Let also $\mathcal{D}(\mathcal{F})$ be the group of diffeomorphisms of M leaving each leaf invariant, and $\mathcal{D}(\mathcal{F},\Sigma)$ be the subgroup of $\mathcal{D}(\mathcal{F})$ consisting of diffeomorphisms fixed on Σ.

Theorem. [1] *The "restriction to Σ map"*

$$\rho\colon \mathcal{D}(\mathcal{F}) \to \mathcal{D}(\Sigma), \qquad \rho(h) = h|_{\Sigma_f},$$

is a locally trivial fibration with fibre $\mathcal{D}(\mathcal{F},\Sigma)$.

This result can be regarded as a "foliated" variant of the well know results by Cerf and Palais on local triviality of restrictions.

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Detecting of chaos by perturbations of degenerate maps

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We study behavior of topological entropy for both multidimensional and one-dimensional perturbations in certain families of chaotic one-dimensional maps. More precisely, let

$$\Phi_\lambda(y_n, y_{n+1}, \ldots, y_{n+m}) = 0, \ n \in \mathbf{Z},$$

be a difference equation of order m with parameter λ. It is asuumed that the non-perturbed operator Φ_{λ_0} is degenerate: it depends on two variables only, i.e.,

$$\Phi_{\lambda_0}(y_0, \ldots, y_m) = \psi(y_N, y_M),$$

where $0 \le N, M \le m$ and ψ is a piecewise monotone piecewise C^2-function. It is also assumed that for the equation $\psi(x, y) = 0$, there is a one-dimensional branch $y = \varphi(x)$ with positive topological entropy. It was proved in [1] that under above assumptions, the perturbed systems are chaotic, and their topological entropy approximates the entropy of the unperturbed map.

We apply this approach in order to detect chaos in certain specific familied of maps as well as in the discrete versions of some PDE's of reaction-diffusion type. We also consider the monotonicity problems for topological entropy in two-parameter families of Lorenz-type maps and we discuss the corresponding bifurcations of the structure of Lorenz attractors.

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Formation of a density bump at the front of a collisionless shock wave during the expansion of a laser plasma

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Recent experiments on thin foils ablation with ultrashort laser pulses revealed unexpected density bumps on the slope of a hot plasma expanding into a cold tenuous ionized medium [1, 2]. A bump propagates at nearly ion-acoustic velocity and is associated with a complex collisionless behavior of particles in the electrostatic shock wave. The shock arises due to the collapse of an ion-acoustic wave packet excited at the beginning of the expansion. The packet is considerably impacted by the flow of high-energy electrons from the dense plasmoid, thus the scenario of free solitons' formation is challenged.

This initial value problem is described by the Vlasov-Maxwell equations. Using 2D3V PIC-simulations for a number of typical plasma parameter sets and density profiles we analyze the dynamics of particles and their phase space evolution, and trace the development of a self-consistent electric field generated meanwhile.

It is found that for the formation and maintenance of the bump, the contributions of both the accelerated ions overtaking the shock and the ions of the background plasma trapped in it are significant. The density value in the bump is consistent with the local density of the background ions, while its velocity is determined by the temperature of the hot electrons. We discuss the conditions for the emergence of a bump, the coordinate of its formation, and the laws of the slow evolution of its velocity and density.

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New research methods of Kupka-Smale diffeomorphisms

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In this paper, we introduce a new invariant of homeomorphisms of a disk with a cascade of periodic orbits - we will call its representation scheme. This invariant distinguishes diffeomorphisms constructed from different sequences of signatures [1,2]. We construct diffeomorphisms of the two-dimensional sphere, which are the results of a twice applied period-doubling bifurcation to a source-sink diffeomorphism, with rotation in one direction and in different directions. The main result is the proof that the schemes of these different forms are not equivalent.

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On quasi-periodic perturbations of Duffing type equations

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We consider the Duffing-Van der Pol equation with a quasiperiodic perturbation

$$\ddot{x} + \alpha x + x^3 = \varepsilon[(p_1 - x^2)\dot{x} + p_2 F(t)], \tag{22}$$

where $\varepsilon > 0$ is a small parameter, $\alpha = \pm 1$, p_1, p_2 are parameters, $F(t) = \sin\omega_1 t \sin\omega_2 t$ is a quasiperiodic function (e.g., $\omega_1 = 1, \omega_2 = \sqrt{5}$).

In the case of $\alpha = 1$, we illustrate the passage of an invariant 3D torus through a resonance zone. The conditions of synchronization are established. In the case of $\alpha = -1$ we mostly investigate the solutions behavior in the neighborhood of the unperturbed separatrix. The conditions of the existence of homoclinic solutions are stated and the matter of complex dynamics is discussed.

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Generation of magnetic field behind the front of a collisionless shock wave during the expansion of a laser plasma

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On the basis of the Vlasov–Maxwell equations in 2D3V space, we numerically study how a hot plasma vaporized from a thin foil by an ultrashort laser pulse expands through a cold and tenuous plasma background. The explosive heating of electrons leads to the formation of a high-energy electron flow and a collisionless shock wave which propagates at nearly ion-acoustic velocity. Soon after the expansion begins, the return electron current supplements the direct one and the magnetic field orthogonal to the simulation plane is generated. Meanwhile the anisotropy of the electron velocity distribution arises behind the shock triggering the Weibel instability and the growth of an in-plane quasi-static inhomogeneous magnetic field.

With the help of PIC-simulations we analyze the evolution of the electron and ion distribution functions and trace the development of electric and magnetic fields in this strongly inhomogeneous plasma. We consider various plasma parameter sets and outline the conditions for the formation of elongated or flattened electron velocity distributions resulting in different spatial structure of the magnetic field. To clarify the saturation of the Weibel instability, we also give results of simulations of the long-term evolution of a homogeneous non-equilibrium bi-Maxwellian plasma and analyze dynamics of the spatial spectrum of the turbulent magnetic field.

The work by A.A. Nechaev and A.V. Mishin was supported by the RFBR grant No. 18-32-01065.

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Cloning of chimera states in large multiplex network of linearly coupled relaxational oscillators with nonstationary inter-layer couplings

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We present, a new phenomenon, the cloning of chimera states, that had been discovered in a large two-layer multiplex system with relaxational oscillators and non-stationary couplings. It is shown that for certain values of strength and time of multiplex interaction, a copy of the chimera state is formed in the initially disordered ensemble. The mechanisms of cloning chimera states are studied. It is established that the phenomenon is not related with synchronization, but arises from the competition of oscillations in the ensembles.

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Isolated cell's response near the boundary of firing mode domain

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Information about any changes in the environment is detected, processed and transferred by neurons, the specific electrically excitable cells of the brain. From the analysis of experimental data obtained in various sensory systems, namely, in auditory and visual systems, the importance of the first spike latency in transmission of information was revealed. The delay in appearance of the electrical response of the neuron is very important for identification of variety of sensory stimuli and carries a greater amount of information about the received stimulus features than subsequent spikes [1].

In the present work we consider the isolated cell's response in dependence on the parameters of the input signal near the boundary of the oscillatory mode domain [2,3].

A comparison of main properties of such response for two models of neuron is presented: for classical Hodgkin-Huxley element and FitzHugh-Nagumo model. The impact of input periodical signal is considered in the presence of additive white Gaussian noise. To reveal the level of cell's sensitivity to noise, we focus on the analysis of both the first spike latency and signal-to-noise ratio dependence on noise intensity.

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Chaotic spatiotemporal dynamics of a chain of bistable maps

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We study the model of a chain of bistable maps with piecewise linear nonlinearity. It‘s show that this model demonstrates two modes of behavior. In the first mode, spatial disorder is realized. In the second mode, spatiotemporal chaos is realized. The transition from the first to the second mode is carried out through a period doubling bifurcation. It‘s shown that the chaotic attractor is the mathematical image of spatiotemporal chaos. Its characteristics, such as the Lyapunov and fractal dimensions, are numerically calculated. Also, the parameter region corresponding to the chaotic attractor was estimated. This area is compared with numerical calculations. Are constructed spatiotemporal plots, which demonstrate spatial disorder and spatiotemporal chaos.

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On estimation of error in approximations provided by Chernoff's product formula

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Introduction. Suppose we have a Banach space $\mathcal{F}$, e.g. $\mathcal{F} = C[0,1]$, $\mathcal{F} = L_2(\mathbb{R})$, $\mathcal{F} = L_p(\mathbb{R}^d)$ for some $p \geq 1$ and $d = 1, 2, 3, \ldots$ Consider a space $\mathcal{L}(\mathcal{F})$ of all linear bounded operators in $\mathcal{F}$, and a one-parameter strongly continuous semigroup (shorter: C_0-semigroup) $V(t)_{t\geq 0}$ in $\mathcal{F}$. This means that $S(t) \in \mathcal{L}(\mathcal{F})$ for each $t \geq 0$, and for each $f \in \mathcal{F}$ we have: $V(0)f = f$, $V(t_1 + t_2)f = V(t_1)V(t_2)f$ for each $t_1, t_2 \geq 0$, and a mapping $[0, +\infty) \ni t \longmapsto V(t)f \in \mathcal{F}$ is continuous. Every C_0-semigroup has an infinitesimal generator, i.e. a linear closed (usually unbounded) operator L which is defined as $L\varphi = \lim_{t\to+0}(V(t)\varphi - \varphi)/t$ for such $\varphi \in \mathcal{F}$ that the limit exists, and the space $D(L)$ of those φ appears to be dense in $\mathcal{F}$. For $\varphi \in D(L)$ we have $V(t)\varphi = \varphi + tL\varphi + o(t)$, i.e. $V(t) = I + tL + o(t)$, which together with $V(t_1 + t_2) = V(t_1)V(t_2)$ explains why the notation $V(t) = e^{tL}$ is widely used.

C_0-semigroups are useful for many reasons, but also because they provide solutions to some class of linear parabolic partial differential equations. In a parabolic equation we have one variable that is different from others and can be considered as time t, let us denote other variables as x, where x may be multi- or even infinite-dimensional. If the coefficients of equation do not depend on time, then the equation can be written in a form $u'_t(t,x) = Lu(t,x)$, where L does not depend on t but may depend on x. Let us provide a very simple example: $x_1, x_2 \in \mathbb{R}, L = \Delta$, then $u'_t(t,x) = Lu(t,x)$ is a heat equation $u'_t(t,x_1,x_2) = u''_{x_1x_1}(t,x_1,x_2) + u''_{x_2x_2}(t,x_1,x_2)$. This class of so-called autonomous evolution equations includes diffusion equation, Scrödinger equation of a closed quantum system and some other equations — remember, that the coefficients can be variable, we only ask them not to depend on t. And in all these cases the solution to Cauchy problem $[u'_t(t,x) = Lu(t,x); u(0,x) = u_0(x)]$ is provided by the C_0-semigroup with generator L as follows: $u(t,x) = (e^{tL}u_0)(x)$, in full analogy with an ODE

$U'(t) = LU(t), U(0) = u_0, U(t) = e^{tL}u_0$. This is the reason why it is interesting to write a formula that expresses $e^{tL}u_0$ explicitly in terms of t, L, u_0, and this is the reason why it is a hard task.

However, there is a way to obtain approximations for e^{tL} via Chernoff's theorem. It says that if we have a C_0-semigroup $(e^{tL})_{t\geq 0}$, and a Chernoff function G for operator L, i.e. a mapping $G\colon [0, +\infty) \to \mathcal{L}(\mathcal{F})$, i.e. another family of bounded operators in $\mathcal{F}$, which is not asked to have a semigroup composition property (this allows to write a simple formula for G in many cases) then under some assumtions we have

$$\lim_{n\to\infty} \|G(t/n)^n f - e^{tL}f\| = 0 \text{ for all } f \in \mathcal{F}, \text{ so } u(t,x) = \lim_{n\to\infty} G(t/n)^n u_0.$$

Assumptions are the following. First, we need to have some $a \in \mathbb{R}$ such that $\|G(t)\| \leq e^{at}$ for all $t \geq 0$. Second, we weed that the mapping $t \longmapsto G(t)f$ is continuous for all $f \in \mathcal{F}$, $G(0) = I$ and $L\varphi = \lim_{t\to+0}(G(t)\varphi - \varphi)/t$ for all φ from the core of L. G is called Chernoff-tangent to L iff the second condition holds, which informally means that $G(t)\varphi = \varphi + tL\varphi + o(t)$ for all φ from some subspace of $\mathcal{F}$ which is large enough to define L. So Chernoff's theorem provides for each G a numerical method to solve the PDE $u'_t(t,x) = Lu(t,x)$. But how accurate this method is and how this accuracy can be explained in terms of G? This talk provides some framework to discuss this question and opens two conjectures. This very short text is written to be self-contained and has limited number of references, wider bibliography appears in a longer paper that will be published later. I only mention two textbooks [1,2] where one can read about the Chernoff theorem and around, plus two papers of mine [3,4] which are devoted to application of Chernoff's theorem to solving Schrödinger equation and parabolic equation with variable coefficients.

Main discussion. On the one hand, for each $t > 0$ and each Chernoff function G for L one can define a function $C_G(t)\colon \mathcal{F} \to c_0$ which maps Banach space $\mathcal{F}$ to Banach space c_0 of all real sequences converging to zero by the rule

$$(C_G(t)f)(n) = \|G(t/n)^n f - e^{tL}f\|,$$

and study properties of $C_G(t)$. Indeed, each portion of the information on the topic that we approach will be a statement describing $C_G(t)$ from some point of view. But on the other hand, this function is not linear and has too many parameters (G, t, f) which may be misleading. The following notion might help to deal with this complexity.

Proposition-Definition 1. Suppose that $w\colon [1,+\infty) \to [0,+\infty)$ and $\lim_{x\to+\infty} w(x) = 0$. Then the set

$$A_w = \Big\{ f \in \mathcal{F} : \|G(t/n)^n f - e^{tL} f\| = o(w(n)) \text{ as } n \to \infty \Big\}$$

is a linear subspace in $\mathcal{F}$. Moreover, if

$$w_2(x) = o(1), w_1(x) = o(w_2(x)) \text{ as } x \to +\infty$$

then

$$A_{w_1} \subset A_{w_2}.$$

Let us call A_w *an approximation subspace,* and let us call the inclusion $A_{w_1} \subset A_{w_2}$ *a hierarchy of approximation subspaces.*

Proof. Given numbers α and β, and vectors f and g from A_w we prove that $h = \alpha f + \beta g$ is a vector from A_w. Indeed: $\|G(t/n)^n h - e^{tL} h\| = \|G(t/n)^n(\alpha f + \beta g) - e^{tL}(\alpha f + \beta g)\| \leq |\alpha| \cdot \|G(t/n)^n f - e^{tL} f\| + |\beta| \cdot \|G(t/n)^n g - e^{tL} g\| = o(w(n)) + o(w(n)) = o(w(n))$. Inclusion $A_{w_1} \subset A_{w_2}$ follows directly from the definition of A_w. $\square$

Let us mention that function w defines A_w uniquely, but the reverse is not true, so hierarchy of subspaces should be simpler than hierarchy of functions converging to zero. Let us state two conjectures which are true in the case $\mathcal{F} = \mathcal{L}(\mathcal{F}) = \mathbb{R}$ and which I believe are true (maybe after some modification) in a general case of infinite-dimensional Banach space $\mathcal{F}$.

Conjecture 1. Let $\left(e^{tL}\right)_{t\geq 0}$ be a C_0-semigroup in a Banach space $\mathcal{F}$, and G is a Chernoff function for operator L, and number $t_0 > 0$ is fixed. Suppose that for all $t \in [0, t_0]$: f is from intersection of domains of operators $G'(t)$ and $G''(t)$, functions $t \longmapsto G'(t)f$ and $t \longmapsto G''(t)f$ are continuous.

Then there exists such a number $C > 0$, that the estimate

$$\left\|G(t/n)^n f - e^{tL} f\right\| \leq \frac{C}{n} \text{ is true for all } t \in [0, t_0) \text{ and all } n \in \mathbb{N}.$$

Conjecture 2. Let $\left(e^{tL}\right)_{t\geq 0}$ be a C_0-semigroup in a Banach space $\mathcal{F}$, and G is a Chernoff function for operator L (recall that this implies $G(0) = I$ and $G'(0) = L$ but says nothing about $G''(0)$), and number $t_0 > 0$ is fixed. Suppose that vector f is from intersection of domains of operators $G'(t)$, $G''(t)$, $G'''(t)$, $G''''(t)$, $G'(t)G''(t)$, $G'(t)^2G''(t)$, $G''(t)^2$ for each $t \in [0, t_0]$, and suppose that if $Z(t)$ is any of these operators then function $t \longmapsto Z(t)f$ is continuous for each $t \in [0, t_0]$.

Then there exists such a number $C > 0$, that for each $t \in [0, t_0)$ and each $n \in \mathbb{N}$ the following inequality holds:

$$\left\| G(t/n)^n f - e^{tL} f + \frac{t^2}{2n} e^{tL} (L^2 - G''(0)) f \right\| \leq \frac{C}{n^2}.$$

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Bifurcations leading to periodic solutions in mathematical model of neural extracellular matrix

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Electrically excitable cells of the brain – known as nerve cells or neurons – form various networks that process and transmit information from one part of the nerve system to another. For a long time it was believed that study of coupling configurations could help to shed light on the main features of such signal transmission. But today it becomes clear that consideration of homogeneous multielement systems that consist of one-type cells, is not enough. To obtain more realistic data, more complicated networks should be investigated. Particularly, the impacts of various structural elements of the environment should also be taken into account. Among them are the glial cells or/and macromolecules of extracellular matrix (ECM) that provide biochemical support of the neurons.

In this work we study the dynamical regimes of ECM mathematical model [1]. Due to both analytical and computer simulations, various quiescent and oscillatory modes was revealed. Based on bifurcational analysis data, various scenaria leading to the birth of periodic orbits in the phase space of the considered system was shown. The change of the neural activity in the presence of quiescent or/and oscillatory modes in dynamics of molecules concentration of ECM is discussed.

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On the influence of the shape of a region on the character of local bifurcations in the Kuramoto-Sivashinsky equation

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In the work the Kuramoto-Sivashinsky equation (KS) in the case of two spatial variables is considered

$$w_t = -v\Delta w - \delta\Delta^2 w - \gamma({w_x}^2 + {w_y}^2), \tag{1}$$

where $w = w(t,x,y), (x,y) \in D = \{(x,y) : x \in [0,l_1],\ y \in [0,l_2]\}$ with boundary conditions

$$w_x|_{x=0,x=l_1} = w_{xxx}|_{x=0,x=l_1} = w_y|_{y=0,y=l_2} = w_{yyy}|_{y=0,y=l_2} = 0. \tag{2}$$

Let $\mu = \frac{l_1}{l_2},\ b = \frac{\nu}{\delta}\left(\frac{l_1}{\pi}\right)^2$ and $\mu > 1$.

Theorem. At $b < 1$ homogeneous equilibrium states $u(t, x) = const$ are stable and they lose their stability if $b > 1$. Moreover, if $b = 1 + \varepsilon$ $(0 < \varepsilon << 1)$, then the problem (1), (2) has a one-parameter family of spatially inhomogeneous solutions depending only on x. The question of local bifurcations can be reduced to the analysis of the normal form

$$z' = z - \frac{1}{12}z^3,\ z = z(s),\ s = \varepsilon t.$$

Now let $b = 1 + \gamma_1\varepsilon,\ \mu = 1 + \gamma_2\varepsilon$, where $\gamma_1, \gamma_2 \in R$. In this case, the question of local bifurcations can be reduced to the analysis of the normal form

$$z_1' = \gamma_1 z - \frac{z_1^3}{12},\ z_2' = (\gamma_1 - \gamma_2)z - \frac{1}{12}z_2^3$$

and at $\gamma_1,\ \gamma_1 - \gamma_2 > 0$ the problem (1), (2) has a one-parameter family of two-mode spatially inhomogeneous solutions depending on x and y.

Let the domain D is an isosceles right triangle

$$0 \le x \le l,\ 0 \le y \le x,$$

and $b = 1 + \varepsilon$. Then, from homogeneous equilibrium states bifurcates a one-parameter family of spatially inhomogeneous solutions depends on x and y.

The main results were published in [1, 2]. Local bifurcations for the generalized KS equation in the case of two spatial variables were studied for periodic boundary conditions, for example, in [3].

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On mixed dynamics of two-dimensional reversible maps with nontransversal heteroclinic cycles

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We consider one parameter families of reversible two-dimensional diffeomorphisms that unfold generally heteroclinic tangencies in symmetric nontransversal heteroclinic cycles. We show that in such families there exist intervals (the Newhouse intervals) containing residual subsets of values of parameter corresponding to coexistence of infinitely many periodic sinks, sources, saddles and elliptic points. Moreover, we show that the closures of the sets of such orbits of different types have nonempty intersections.

Formation of hyperchaos after secondary Neimark-Sacker bifurcation

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The chaos generated by nonlinear dynamical systems was discovered in the first half of the last century. Chaotic dynamics is a fundamental property of nonlinear systems, which was revealed in almost all areas of science: physics, radiophysics, optics, biophysics, Josephson contact dynamics, neurodynamics, mechanics, chemistry, etc. [1-2]. During this time, a lot of research into the properties of chaos was conducted, scenarios of its birth were discovered, and a large number of applications were developed where dynamic chaos turned out to be very productively used.

Chaos is most reliably diagnosed using Lyapunov exponents. According to the number of positive Lyapunov exponents hyperchaos can be classified, when there are two or more positive exponents in the spectrum. Today, the properties of dynamic chaos with one positive Lyapunov exponent are most fully described. Also a sufficiently large number of models with hyperchaos were presented. However, the scenarios of the formation of hyperchaos are unsolved problem. The main scenario which is described in papers [3-5] is associated with riddling bifurcation.

In the main focus of the present work a case when hyperchaos occurs after secondary Neimark-Sacker bifurcation. We will consider different dynamical models with secondary Neimark-Sacker bifurcation, and make numerical simulations, where formation of hyperchaos is observed. As a main tool for investigation we will use Lyapunov exponents and phase portraits in Poincaré section.

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Towards the appearance of chaotic dynamics in the Hénon map

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We study the scenarios of the appearance and break-down of homoclinic attractors in two-dimensional Hénon map. Recall, that under homoclinic attractor we mean strange attractor which contains one saddle orbit with its unstable manifold [1]. Also we show that the emergence of chaotic dynamics in two-dimensional Hénon map can appear before the transition to chaos via Feigenbaum cascade for the stable fixed point.

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Transitions between regimes in ensembles of coupled neurooscillators in application to epilepsy modeling

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Epileptic seizures can be considered as result of bifurcations in the brain network, leading to emergence of highly synchronous collective dynamics in large ensembles of cortical, thalamic and hippocampal cells [1]. While this phenomenon was clearly found for many types of epilepsy in different brain structures and circuit, the detailed study is still imperfect. The main reason is brain complexity and lack of possibility to measure enough signals simultaneously and without risk to damage generating cells (actually, hundreds of cells have to be measured from not only neocortex but deeper brain structures also, what is still impossible even in animals). Therefore, bifurcation mechanisms in the brain are still studied based on mathematical models constructed specially for different particular cases, e. g. in the case absence epilepsy see [2]. In this framework, it is usually stated that changes in coupling lead to changes in brain activity, not changes in self properties of individual cells. However, this fact cannot be easily tested.

Here we propose a number of simple oscillatory stochastic models, usually constructed by modification well known two and 3-dimensional dynamical systems (van der Pol oscillator, FitzHugh–Nagumo system, Morris–Lecar system, Hindmarsh–Rose system, Rossler system). Due to the specially proposed way of coupling ensembles of these oscillators can demonstrate switching between three main regimes: high amplitude dynamics with a single main time scale (high frequency and low frequency) and low amplitude noisy dynamics without any main timescale. The first two regimes may characterize the epileptiform activity (different stages and different types of epileptic activity) and the third one characterizes the normal electroencephalogram (or signals of local field potentials). Transition between regimes can be achieved both by change of parameters of individual subsystems and by modifying the coupling coefficient. The similar approach was previously applied by us for modeling absence seizures [3], and for limbic seizures [4].

The proposed models give a possibility to test different methods of coupling estimation, including Granger causality, transfer entropy, partial directed coherence and methods of phase dynamics modeling (for regimes with a main time scale), and to understand whether these approaches can reveal the actual reason of regime change in operator of evolution, or they only response to functional coupling change due to difference in statistical properties of the signals in different regimes.

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