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BOOK OF ABSTRACTS

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A method for prediction and computation of nonlinear modes in NLS-type models

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We present a method for prediction and computation of stationary nonlinear modes for systems of equations of Nonlinear Schrödinger (NLS) type. These stationary nonlinear modes are described by a system of nonlinear ODE of the form

$$\mathbf{u}_{xx} + \mathbf{A}(x)\mathbf{u} - \mathbf{B}(\mathbf{u}, \mathbf{u}; x)\mathbf{u} + \mathbf{h}(x) = 0$$

Here $\mathbf{u}(x)$ is an n-component real vector function, $\mathbf{u}(x) = \text{col } (u_1(x), \ldots, u_n(x))$; $\mathbf{A}(x)$ is an $n \times n$ matrix function of x; $\mathbf{B}(\mathbf{a}, \mathbf{b}; x)$ is a diagonal $n \times n$ matrix where the diagonal entries $B_{k,k}(\mathbf{a}, \mathbf{b}; x)$, $k = 1, \ldots, n$, are bilinear forms of n-component vectors $\mathbf{a}$ and $\mathbf{b}$ with the coefficients depending on x; $\mathbf{h}(x)$ is an n-component vector function of x. We prove that under some conditions the solutions of this system generically have singularities on the real axis and, consequently, cannot describe a profile of a nonlinear mode. Therefore the approach consists in detecting of *bounded* (singularity-free) solutions of the system and their thorough analysis. The method (called *the method of excluding singular solutions*) is illustrated by the example of the Lugiato-Lefever equation that describes the nonlinear waves in an optical cavity. Existence of some novel nonlinear modes (solitons) is reported.

Application of Dynamical Systems to the Study of Asymptotic Classification and Asymptotic Behavior of Solutions to Nonlinear Higher-Order Differential Equations

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Consider the equation

$$y^{(n)} = p(x, y, y', \ldots, y^{(n-1)})|y|^k \operatorname{sign} y, \;\; n > 4, \;\; k > 1. \tag{1}$$

New results are proved on asymptotic behavior of blow-up and Kneser (see [1, Definition 13.1]) solutions to this equation. To prove the results the equation reduce to a dynamical system on an (n - 1)-dimensional compact sphere (see [6]). We study the behavior of the trajectories of this system corresponding to constant-sign parts of solutions of (1). By this method the asymptotic classification of solutions to (1) with $n = 3$, 4 was also obtained (see [7]).

Theorem 1. *Suppose $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\ldots,y_{n-1}}(\mathbf{R}^n)$ and $p \to p_0 > 0$ as $x \to x^*, y_0 \to \infty, \ldots, y_{n-1} \to \infty$. Then for any integer $n > 4$ there exists $K > 1$ such that for any real $k \in (1, K)$, any solution to equation (1) tending to $+\infty$ as $x \to x^* - 0$ has power-law asymptotic behavior, namely*

$$y(x) = C(x^* - x)^{-\alpha}(1 + o(1)) \tag{2}$$

with

$$\alpha = \frac{n}{k-1}, \quad C^{k-1} = \frac{1}{p_0} \prod_{j=0}^{n-1} (j+\alpha)\,. \tag{3}$$

Theorem 2. *Suppose* $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\dots,y_{n-1}}(\mathbf{R}^n)$ *and* $(-1)^n\, p \to p_0 > 0$ *as* $x \to \infty$, $y_0 \to 0, \dots, y_{n-1} \to 0$. *Then for any integer* $n > 4$ *there exists* $K > 1$ *such that all Kneser solutions to equation (1) with any real* $k \in (1, K)$ *tend to zero with power-law asymptotic behavior, namely*

$$y(x) = C(x-x^*)^{-\alpha}(1+o(1)),\ x \to \infty, \tag{4}$$

with some x^* *and* α, C *given by (3).*

Earlier it was proved that for $n = 3, 4$ all blow-up and Kneser solutions to equation (1) have the power-law asymptotic behavior (see [2], [3]). It was also proved for equation (1) with $(-1)^n\, p \equiv p_0 > 0$ for sufficiently large n (see [4]) and for $n = 12, 13, 14$ (see [5]) that there exists $k > 1$ such that equation (1) has a solution with non-power-law behavior, namely

$$y(x) = (x-x^*)^{-\alpha}\ h(\log{(x-x^*)}), \tag{5}$$

where h is a positive periodic non-constant function on $\mathbf{R}$. For blow-up solutions see also [5]–[6].

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Equivariant simple singularities and their classification

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Given two linear representations of a group G on $\mathbb{C}^n$ and on $\mathbb{C}$, we call a function $f\colon \mathbb{C}^n \to \mathbb{C}$ *equivariant* with respect to the given representations if for all $\lambda \in G,\ z \in \mathbb{C}^n$ the condition $f(\lambda \cdot z) = \lambda \cdot f(z)$ holds. Similar definitions can be given for holomorphic function germs at 0 and for zero-preserving biholomorphic automorphisms of $\mathbb{C}^n$ and their germs at 0.

The group $\mathcal{D}_n^{GG}$ of equivariant biholomorphic germs $\Phi\colon (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ acts on the space $\mathcal{O}_n^{GG}$ of equivariant function germs $f\colon (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$. The space $\mathcal{O}_n^{GG}$ is split into orbits of this action. The same is true for all spaces $j_r\mathcal{O}_n^{GG}$ of r-jets at 0 of germs from $\mathcal{O}_n^{GG}$. The orbit $\mathcal{D}_n^{GG}(j_r g) \subset j_r\mathcal{O}_n^{GG}$ is said to be *adjacent* to the orbit $\mathcal{D}_n^{GG}(j_r f)$ if any neighborhood of some (and therefore any) point in $\mathcal{D}_n^{GG}(j_r f)$ intersects with $\mathcal{D}_n^{GG}(j_r g)$.

A germ $f \in \mathcal{O}_n^{GG}$ is called *equivariant simple* with respect to the given representations of the group G on the source and target if for all $r \in \mathbb{N}$ the orbit $\mathcal{D}_n^{GG}(j_r f) \subset j_r\mathcal{O}_n^{GG}$ has only a finite number of adjacent orbits, and this number is bounded from above by a constant M independent of r.

Two germs $f, g \in \mathcal{O}_n^{GG}$ are called *equivariant right equivalent* if there exists a germ $\Phi \in \mathcal{D}_n^{GG}$ such that $g = f \circ \Phi$. There exists a general problem to classify all equivariant simple function germs in $\mathcal{O}_n^{GG}$ with a critical point at $0 \in \mathbb{C}^n$ (for a given group G and a pair of its linear representations on $\mathbb{C}^n$ and on $\mathbb{C}$) up to this equivalence relation. This problem is a natural generalization of a similar problem for the non-equivariant case that has been solved by V.I.Arnold in [1]. Particular cases of the general problem for $G = \mathbb{Z}_2$ are treated in [2] and [3].

In the talk I will present some recent results related to the general problem for the case of a cyclic group $G = \mathbb{Z}_m$. In particular, a complete classification of equivariant simple singular function germs of two and three variables for the group $G = \mathbb{Z}_3$ will be given. Some of the presented results can be found in [4], [5].

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Chimera states in a modified Kuramoto model with inertia

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We consider dynamics of the modified Kuramoto model with inertia

$$\beta\ddot{\phi}_i + \dot{\phi}_i = \omega_i - \mu\sin\phi_i + \frac{1}{N}\sum_{j=1}^{N} k_{ij}\sin(\phi_j - \phi_i), \qquad i = \overline{1, N}, \tag{6}$$

where $\beta > 0, \quad \mu > 0, \quad k_{ij} = k\varepsilon_{ij}, \quad \omega_i$ - natural frequency of the i - th oscillator. In the case $\beta = 0, \quad \mu = 0$ the system (6) takes the form of classical Kuramoto model. In contrast to classical Kuramoto model the uncoupled system (6) for $\beta = 0, \quad k_{ij} \equiv 0$ is determined by the first-order phase equation $\dot{\phi}_i = \omega_i - \mu\sin\phi_i$. The case of "spatially" homogeneous system (6) for $k_{ij} = k, \quad \omega_i = \omega, \quad i, j = 1, 2, \ldots, N$ which has $2D$ invariant manifold $M(2) = \{\varphi_i = \varphi, \quad i = 1, 2, \ldots, N\}$ is considered in the talk. The main result of this work is that for some parameters region there exists an invariant $4D$ manifold $M(4) = \{\varphi_1 = \varphi_2 = \cdots = \varphi_m = \varphi, \quad \varphi_{m+1} = \varphi_{m+2} = \cdots = \varphi_N = \psi\}$ of the system (6) which does not coincide with $M(2)$. It contains a stable periodic orbit P, a variable φ along which oscillates, and the variable ψ rotates (Fig.1).

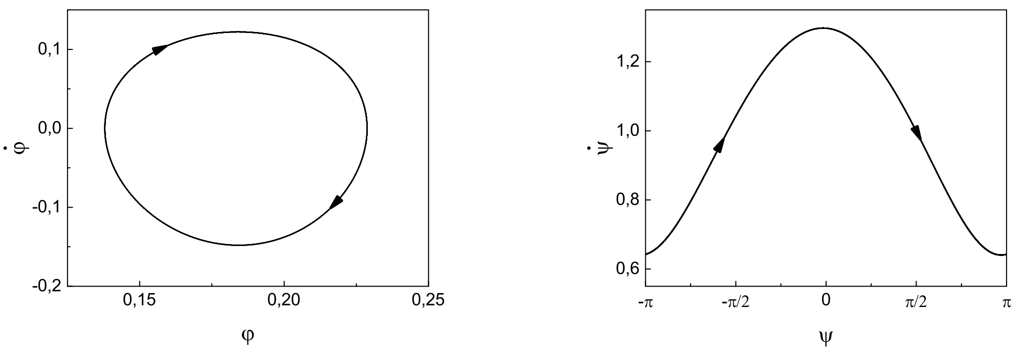

Figure 1: Projections of periodic orbit P of the system (6) on the planes $(\varphi, \dot{\varphi}), (\psi, \dot{\psi})$ manifesting the chimera state.

The synchronous variables $\varphi_1 = \varphi_2 = \cdots = \varphi_m$ for $\frac{m}{N} < q < \frac{1}{2}$ correspond to the "chimera" (in a homogeneous system (6) the periodic orbit P is inhomogeneous: φ differs from ψ). Sufficient conditions for the existence of the manifold $M(4)$ are obtained using comparison systems. In addition, for the system (6) sufficient conditions for complete synchronization are analytically studied.

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Synchronization phenomena and crowd dynamics on a wobbly bridge

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In the talk we discuss the "locking-in" and "pulling-in" phenomena discovered by Van der Pol in 1920-th. As well, our results on the Huygens clocks synchronization (1660-th) are presented. These phenomena have a direct bearing on the problem of synchronization of pedestrians on a wobbly bridge. This problem is the main goal of this talk. We first introduce an inverted pendulum model of pedestrian balance, and use it in the equations of pedestrians-bridge interaction of the form

$$\ddot{x}_i + f_i(x_i, \dot{x}_i) = -\ddot{y}, \quad \ddot{y} + 2h\dot{y} + \Omega^2 y = -r\sum_{i=1}^{n} \ddot{x}_i,$$

where x_i is lateral coordinate of i-th pedestrian, y is bridge coordinate and the function f_i defines the pedestrian movement. We derive explicit analytical conditions under which phase locking and bridge wobbling appear in the system when the crowd size exceeds a threshold value.

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Breather chimeras in the system of phase oscillators

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We study formation of chimeras in a one-dimensional medium of identical oscillators with nonlinear coupling. This coupling crucially depends on the local order parameter measuring the level of synchrony: the coupling promotes synchrony for asynchronous states and breaks synchrony if it is strong [1]. As a result, spatially homogenous state in this medium is that of partial synchrony. To study the evolution of this state we formulate the problem in terms of the local complex order parameter, which describes local level of synchrony, and formulate the system of partial differential equations for this quantity [2]. This allows us to formulate the problem of inhomogeneous states as the pattern formation one. First, we construct stationary chimeras and explore their linear stability properties. Next, based on numerical modeling, we show that within a certain range of parameters, such structures can evolve into periodically varying long-lived chimera states (breather-chimeras), or, for other values of the parameters, turn into more complex regimes with irregular behavior of the local order parameter (turbulent chimeras) [3].

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A task for a student's practical work. Qualitative and numerical studies of ordinary differential equations

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Dedicated to the memory of N. S. Bakhvalov

Nikolai S.Bakhvalov was in search of a task for the 4th year students of the Faculty of Mechanics and Mathematics of the Moscow State University. Such a task, that students might perform during the lesson in the computer class, after listening to the course of ordinary differential equations and studying the course "Computational methods" [3]. Moreover, such a task that is impossible or very difficult to perform without a computer. Alexey F. Filippov gave him a list of calculating tests of the 50th years, and Nikolai Sergeyevich chose one of them. This task [1, 2] was formulated as follows: to check whether periodic solutions for a given two-dimensional system of differential equations exist. If periodic solutions exist, then it is required to find their periods. This task was supported by the head of the Department of Mathematics of the Faculty of Mechanics and Mathematics of the Moscow State University academician Andrei Nikolaevich Kolmogorov, who noted that it may be useful for solving the 16th Hilbert's problem. To deliver various tasks to students, N.N.Chentsova selected 12 systems of ordinary differential equations, depending on the real parameter. These systems have been partially investigated in various sources. The values of the parameters for which there is certainly a periodic solution were chosen by N.N.Chentsova on the basis of the theorems known earlier, her qualitative studies and numerically by computer. Further, the require to build a phase portrait was added, i.e.

1. Please, find all the singular points of the system (in explicit form and numerically by a computer using the Newton's method). **2.** Please, determine the type of each finite singular point. Please find the eigenvectors and eigenvalues (the Jacobian matrix at the singular point is calculated in explicit form and numerically by a computer using the optimal step choice).

3. Please, draw by hand sketches of orbits of trajectories in the neighbourhood of each finite singular point, using point 2. **4.** Please, approximately by a computer (using the optimal step choice of Runge-Kutta methods) calculate the trajectories you have selected and graphically depict them on the computer screen (including periodic solutions and separatrices of the saddles).

Further, the require to investigate bifurcations on parameter was added.

Task list:

1. $\begin{cases} x' = y, \\ y' = -x + d \cdot (1 - x^2) \cdot y, \end{cases}$ 2. $\begin{cases} x' = x + y - d \cdot x^3, \\ y' = -x + y - y^3, \end{cases}$

3. $\begin{cases} x' = y + x^3 - d \cdot x^5, \\ y' = -x + y^3 - d \cdot x^5, \end{cases}$ 4. $\begin{cases} x' = x + y - x \cdot (x^2 + x \cdot y + y^2), \\ y' = -x + y - d \cdot y \cdot (x^2 + x \cdot y + y^2), \end{cases}$

5. $\begin{cases} x' = y, \\ y' = d \cdot y - x - x^2 \cdot y, \end{cases}$ 6. $\begin{cases} x' = -y + d \cdot (x^2 + y^2 - d) \cdot x, \\ y' = x + d \cdot (x^2 + y^2 - d) \cdot y, \end{cases}$

7. $\begin{cases} x' = y, \\ y' = -0,2 \cdot d^2 - d \cdot x + x^2 + x \cdot y, \end{cases}$ 8. $\begin{cases} x' = d \cdot x + y + x^2 + y^2, \\ y' = d \cdot y - x + x^2 + 2 \cdot y^2, \end{cases}$

9. $\begin{cases} x' = d \cdot x + y + x^2 + 2 \cdot y^2, \\ y' = d \cdot y - x + 2 \cdot x^2 + 5 \cdot y^2, \end{cases}$ 10. $\begin{cases} x' = y, \\ y' = d \cdot y - x - y^3, \end{cases}$

11. $\begin{cases} x' = y, \\ y' = -d \cdot x + y - x^2 \cdot y - x^3, \end{cases}$ 12. $\begin{cases} x' = y, \\ y' = d \cdot y - x - x^2 \cdot y + x^3. \end{cases}$

The values of the parameter d:

variant	student	d1	d2	d3	variant	student	d1	d2	d3
					7	7	-0.1	0.1	2.2
1	1	-1.0	0.1	4.0	7	13	-0.05	0.3	3.1
1	24	-2.5	0.35	3.6	7	30	0.02	0.2	2.0
2	2	0.1	2.4	8.3	8	8	-0.01	0.01	1.0
2	23	0.08	1.2	7.1	8	14	-0.007	0.007	0.4
2	35	0.04	5.8	6.8	8	29	-1.0	0.009	0.05
3	3	-0.5	0.5	3.4	9	9	-0.01	0.01	1.0
3	22	-1.2	0.9	4.6	9	15	-0.005	0.005	0.5
3	34	-2.0	1.5	5.2	9	28	-1.0	0.005	0.05
4	4	0.1	3.7	10.0	10	10	-1.0	1.0	3.0
4	21	-0.1	2.3	7.1	10	16	-2.0	1.5	3.3
4	33	-0.5	4.1	8.7	10	27	-0.5	0.5	5.0
5	5	-1.1	0.2	4.7	11	11	-3.0	1.0	6.0
5	20	-2.5	0.8	10.0	11	17	-2.0	-0.02	2.0
5	32	-3.0	0.5	7.4	11	26	-0.01	0.1	3.5
6	6	-1.0	2.0	5.4	12	12	-0.01	0.01	1.0
6	19	-2.2	3.3	6.5	12	18	-0.004	0.004	0.4
6	31	-3.0	4.2	9.0	12	25	-1.0	0.005	0.09

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Stochasticity of the dynamic system. Sufficient conditions for testing

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Dedicated to the memory of L.P.Shilnikov

The autogenerator with the tunnel diode is given in [1] by the following system of differential equations (compare with systems in [2]):

$$\begin{cases} \dot{x} = y - \delta \cdot z, \\ \dot{y} = -x + 2 \cdot \gamma \cdot y + \alpha \cdot z, \\ \mu \dot{z} = x - f(z). \end{cases} \tag{1}$$

In [4, 5] the function $f(z)$ is given as a piecewise linear function:

$$f(z) = \begin{cases} 2 \cdot (z-1), & z \geq 2/3, \\ -z, & -2/3 \leq z \leq 2/3, \\ 2 \cdot (z+1), & z \leq -2/3. \end{cases} \tag{2}$$

Theorem. For all parameters $\alpha, \beta, \gamma, \delta \in R$ of a non-empty neighborhood Δ containing the point $(\alpha = 1/4, \gamma = 1/4, \delta = 1/2)$ the Poincare map π of the plane $z = 2/3$ into itself contains a smooth Smale's horseshoe with k connected components ($k = 4$) (see the definition in [3], [10]) and is stochastic in the sence of [3-8] as $\mu \to +0$.

In particular, there exists a π-invariant hyperbolic set Ω, which is a Cantor perfect set. Ω contains k^n periodic points of period n (see pictures in [9]).

For the proof (see [5]), the conditions of uniform hyperbolicity of the of the Poincare map $\pi : (x, y) \to (f(x, y), g(x, y))$

$$\left|\left|\frac{\partial f}{\partial x}\right|\right| < 1, \quad \left|\left|\left(\frac{\partial g}{\partial y}\right)^{-1}\right|\right| < 1,$$

$$\left|\left|\left(\frac{\partial g}{\partial y}\right)^{-1} \cdot \frac{\partial g}{\partial x}\right|\right| \cdot \left|\left|\frac{\partial f}{\partial y}\right|\right| < \left[1 - \left|\left|\frac{\partial f}{\partial x}\right|\right|\right] \cdot \left[1 - \left|\left|\left(\frac{\partial g}{\partial y}\right)^{-1}\right|\right|\right],$$

on each connected component are tested, using the analytic representation of the part of the solution, which is inside of one of the three domains given by the partition (2) and the asymptotic expansions as the parameter $\mu \to +0$. The geometric configuration of the initial horseshoe decomposition into k connected components and the existence of foliations are also checked.

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Scattering maps and Arnold diffusion in Hamiltonian systems for complete families of perturbations

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We prove that for any non-trivial perturbation depending on any two independent harmonics of a pendulum and a rotor there is global instability. Similar results apply for one pendulum and two rotors. The proof is based on the geometrical method and relies on the concrete computation of several scattering maps. A complete description of the different kinds of local scattering maps taking place as well as the existence of piecewise smooth global scattering maps is also provided. This is a joint work with Rodrigo G. Schaefer, also from UPC.

Topological states of matter as bifurcations of trajectories of Schrödinger equation in periodic potential

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Topological physics has been one of the most rapidly developing fields in the last decade. The main idea is to study connections of the vector bundle generated by a gauge field for the trajectories of a dynamical system determined by Schrödinger equation:

$$i\hbar \frac{d\psi(x)}{dt} = H\psi(x)$$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + U,\ U\psi(x+R) = U\psi(x)$$

The periodicity of the potential U demands the solution to have a form, $e^{i\pi k u_k(x)}$. Hence the spectrum of the Hamiltonian is also parametrised by k, which can be associated with the gauge field. For a fermionic spinless system, multiple (degenerate) eigenvalues are not allowed, but in numerical study such a situation occurs and can be protected by crystal symmetries which are used in the simulation of the periodic potential. This problem is resolved by introducing a spin degree of freedom. In this case electrons in a crystal show a spin dependent behavior such as spin textures, the spin Hall effect, and spin ordering (magnetism). The development of a predictive analysis of such properties in real compounds is one of the most important goals for theoretical physics. Here we introduce mathematical tools that physicists use for attacking these issues; emphasizing the concept of Berry curvature on the Hamiltonian spectrum and the Kubo computational formalism for the calculation of certain transport properties (and its connection with degenerate points in spectrum). The spin dependent properties mentioned above are essentially a bifurcation of the quantum system from the non-ergodic to ergodic regime and thus can be studied in the dynamical system theory environment in combination with topological tools used now and we aim to discuss the open questions of this field.

On the local equilibrium of equations system of Godunov-Sultangazin

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The main equation of the kinetic theory is the Boltzmann equation [1]. We consider the discrete kinetic system of the Godunov – Sultangazin equations [2,5]:

$$\begin{gathered}\partial_t u + \partial_x u = \frac{1}{\varepsilon}(v^2 - uw) \\ \partial_t v = -\frac{2}{\varepsilon}(v^2 - uw) \\ \partial_t w - \partial_x w = \frac{1}{\varepsilon}(v^2 - uw) \\ v(0) = v^0,\ u(0) = u^0,\ w(0) = w^0.\end{gathered} \tag{7}$$

This system describes a gas consisting of three groups of particles moving along a straight line where $u(x,t), v(x,t), w(x,t)$ are densities of particles. The Godunov – Sultangazin system is a model system for the Boltzmann kinetic equation. There are another models such as Carleman, Broadwell [3,4]. The local equilibrium of solutions of the Cauchy problem with bounded energy and periodic initial conditions are investigated. Under certain general assumptions, the solutions of the problem tends to equilibrium state.

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On behavior of oscillating solutions to the second-order Emden–Fowler type differential equations near the boundaries of domain

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Consider the second-order Emden–Fowler type differential equation

$$y'' + p(x,\, y,\, y')\, |y|^k \operatorname{sgn} y = 0,\ k > 0,\ k \neq 1, \tag{8}$$

with the positive potential $p = p(x, u, v)$ defined on $\mathbb{R} \times \mathbb{R}^2$. Assume that the potential p is continuous in x and Lipschitz continuous in u, v and satisfies inequalities:

$$0 < m \le p(x, u, v) \le M < +\infty. \tag{9}$$

It was proved [1] that all nontrivial maximally extended solutions and their first derivatives to equation (8) are oscillating near the boundaries of domain. Zeroes x_j of solutions and zeroes x'_j of their first derivatives alternate, i. e.

$$\ldots < x_{j-1} < x'_j < x_j < x'_{j+1} < \ldots, \quad j \in \mathbb{Z}.$$

I.T. Kiguradze and T.A. Chanturia [2] described the asymptotic behavior of all solutions to equation (8) in the case $p = p(x)$. The results on asymptotic classification of maximally extended oscillating solutions to the third- and the fourth-order similar differential equations are given by I.V. Astashova in [3, Chapter 6] and [4, 5]. Moreover, it was proved [2] that if $p = p(x)$ is a positive locally integrable function of locally bounded variation, then any nontrivial right-maximally extended solution to equation (8) is proper, i. e. it is defined in a neighborhood of $+\infty$ and $\sup\{|y(s)| : s \ge t\} > 0$.

Using the methods described in [3] we investigate the behavior of solutions to equation (8) near the boundaries of domain. The assumption that function p be of locally bounded variation is essential (see [6]). For $k > 1$ an example is given of a continuous function $p = p(x)$ satisfying inequalities (9) such that there exists a solution to (8) with a resonance asymptote $x = x^*$ ($\overline{\lim\limits_{x\to x^*}}\, y(x) = +\infty$, $\underline{\lim\limits_{x\to x^*}}\, y(x) = -\infty$). Step by step we construct the continuous function p and the oscillating solution $y(x)$ to equation (8). On each step we also estimate the distance between consecutive zeroes $x_{j+1} - x_j$.

Furthermore, we obtain the sufficient conditions on the function $p = p(x)$ guaranteeing that all nontrivial maximally extended solutions to equation (8) are defined on the whole axis.

Theorem. *Suppose $k \in (0, 1) \cup (1, +\infty)$, $p = p(x)$ is a continuous function of globally bounded variation satisfying inequalities (9). Then for any nontrivial maximally extended solution $y(x)$ to (8) the following finite positive limits exist:* $\lim\limits_{j\to\pm\infty} |y'(x_j)|$, $\lim\limits_{j\to\pm\infty} |y(x'_j)|$, $\lim\limits_{j\to\pm\infty} (x_{j+1} - x_j)$.

Moreover, we prove that the assumption of globally bounded variation for function $p(x)$ is essential for the existence of the finite positive limits $\lim\limits_{j\to\pm\infty} |y'(x_j)|$, $\lim\limits_{j\to\pm\infty} |y(x'_j)|$, $\lim\limits_{j\to\pm\infty} (x_{j+1} - x_j)$. An example of a continuous function $p(x) > 0$ (satisfying inequalities (9) but not of globally bounded variation) is given (see [6]) such that there exists an unbounded proper solution $y(x)$ such that $\lim\limits_{j\to+\infty} |y'(x_j)| = \lim\limits_{j\to+\infty} |y(x'_j)| = +\infty$. Also an example of a continuous function $p(x) > 0$ (satisfying inequalities (9) but not of globally bounded variation) is given [6] such that there exists a nontrivial proper oscillating solution tending to zero with its first derivative as $x \to +\infty$.

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Miquel dynamics on 2x2 periodic circle patterns

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The circle patterns were introduced to suggest a way to uniformize graphs on surfaces: embedded graphs with quadrilateral faces such that each face is inscribed in a circle. We will describe a dynamical system on the space of circle patterns with $\mathbb{Z}^2$ square grid combinatorics: the *Miquel dynamics.* Its definition is based on a classical theorem of plane geometry: Miquel six circles theorem. There is a conjecture stating that this system is integrable. We consider its restriction to the 2x2 periodic circle patterns and present some results partly confirming its integrable nature. Namely we show that the complexification of the space under question is a fibration by invariant elliptic curves (canonically realized as binodal quartics in $\mathbb{CP}^2$), and on each elliptic curve the dynamics acts by a shift.

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Structurally stable systems and topology of ambient manifold

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Report is devoted to exposition of results concerning of interrelation between dynamics of structurally stable system and topology of ambient manifolds.

For systems with regular dynamics we will discuss what role heteroclinic intersections of stable and unstable invariant manifolds of saddle equilibrium states of flows and saddle periodic points of cascades play in topological structure of ambient manifolds.

For cascades with chaotic dynamics we will show how dimension and type of basic sets belonging to nonwandering set are connected with topology of ambient manifolds.

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Dynamics of solenoidal basic sets

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An A-diffeomorphism $f : M^n \to M^n$ is called *Smale-Vietoris* (in short, SV-diffeomorphism) if M^n contains a sub-manifold $\mathcal{B}^n \overset{\text{def}}{=} S^1 \times D^{n-1}$ (a *basic* sub-manifold) such that the restriction $f|_{\mathcal{B}^n} \overset{\text{def}}{=} F$ satisfies the following conditions:

1. The map F is of the type $F(t,z) = (g(t), w(t,z)),\ t \in S^1,\quad z \in D^{n-1}$, where $g : S^1 \to S^1$ is a preserving orientation d-cover mapping, $d \geq 2$;
2. For any $t \in S^1$, $w|_{\{t\}\times D^{n-1}} : \{t\} \times D^{n-1} \to \mathcal{B}^n$ is a uniformly contracting transformation $\{t\} \times D^{n-1} \to int\left(\{g(t)\} \times D^{n-1}\right)$; this means that there are $0 < \lambda < 1$, $C > 0$ such that $diam\ (F^k(\{t\} \times D^{n-1})) \leq C\lambda^k diam\ (\{t\} \times D^{n-1}),\ \forall k \in N$.

The intersection $\cap_{k\geq 0} F^k(\mathcal{B}^n) \stackrel{\text{def}}{=} Sol$ is a (topological) solenoid. The non-wandering set belonging to the basic sub-manifold is in the solenoid. By Spectral Decomposition Theorem, the non-wandering set is a union of piecewise disjoin transitive closed sets called basic sets. Recall that a basic set if *nontrivial* if it is not an isolated periodic orbit. The following proposition describes basic sets in $Sol \subset \mathcal{B}^n$.

Theorem. Given any SV-diffeomorphism $f : M^n \to M^n$, the non-wandering set $NW(f) \cap \mathcal{B}^n$ belonging to the basic sub-manifold has a unique nontrivial basic set Λ that is

- either a one-dimensional expanding attractor, and $\Lambda = Sol$ in this case, or
- Λ is a zero-dimensional basic set, and in this case the non-wandering set $NW(f) \cap \mathcal{B}^n$ consists of Λ, and finitely many (non zero) sink periodic orbit, and finitely many (possibly zero) isolated saddle periodic orbits.

The both cases above are realized.

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Bacterial drift velocity along the gradient of the chemoattractant concentration

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The study of the microorganisms chemotaxis phenomena is actively developing at present [1-4]. Recent experimental observations revealed several types of random walks, that determine the characteristics of the various bacteria motor reaction in response to chemical stimulus in the environment. The most well-known and well-studied example is the strategy of the *Escherichia coli* movement – rod-shaped bacteria, which is widely spread in the lower intestines of the warm-blooded animals. The movement of this bacteria consists of a series of smooth movements (with duration of a few seconds), interspersed by short turns (with duration about a tenth of a second). The distribution of angles between the directions of motion before and after the turn shows a single peak corresponding to approximately 70°. The efficiency of chemotaxis of bacteria using a strategy that does not change rotation angle in average, were investigated using the approach proposed by de Gennes [2, 3], and was generalized to the case of the strategy with two successive rotation angles, one of which corresponds to turn on the random angle with the mean value of 90° [4]. The consideration of this particular case allowed us to show that the second considered strategy is more effective than the strategy of a movement, for example, *E. coli*, as it provides a higher drift velocity of the bacteria.

Recent experimental studies have shown that a random walk of the bacterium *Vibrio alginolyticus* can be represented in the form of alternations between the two different average values of the angles, one of which depends on the size of the bacteria, and the other is equal to the average of 172° [5]. It should be noted that in this case the calculation of the drift velocity becomes non-trivial due to the necessity to take into account the history of the bacteria movement. Following the ideas of de Gennes, we analytically calculated the drift velocity for this case. The obtained results show a complete match with the special cases considered in [3,4]. It is noteworthy that the studied model allows to take into account the degree cell populations heterogeneity and to relate differences in the cells behavior in the population with their size.

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Behavior of spatially inhomogeneous solutions of nonlinear optical system

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Consider equation with a deviation of spatial variable

$$\frac{\partial u}{\partial t} + u = \varepsilon \frac{\partial^2 u}{\partial x^2} + K \sin u(t, x - h) \tag{1}$$

and periodic boundary conditions

$$u(t, x + 2\pi) \equiv u(t, x). \tag{2}$$

This problem was introduced to describe some nonlinear effects in optic [1]. Here $0 < \varepsilon \ll 1$, $K \in R$. The parameter h, which characterizes the deviation of the spatial variable (rotation of the field by an angle h), is close to rationally multiples of 2π, i.e. for some relatively prime numbers m_1 and m_2 we have relation

$$h = 2\pi \frac{m_1}{m_2} + \mu,$$

where μ is another one small parameter: $0 < \mu \ll 1$.

Let u_0 be a homogeneous equilibrium state (1), (2): $u_0 = K \sin u_0$. We'll study behavior of solutions (1), (2) in some neighborhood of u_0.

The most interesting case is a situation when parameter $p = K \cos u_0$ is clode to -1. This mean that for sufficiently small ν $(0 < \nu \ll 1)$ $p = -1 - \nu$. So, problem contains three small parameters: ε, μ and ν. It is shown that their ratio is very important and has a significant impact on both the results and the course of research.

In the critical case real parts of infinite number of roots of characteristic equation

$$\lambda_k = -1 + p \exp(-ikh) - \varepsilon k^2 \quad (k = 0, \pm 1, \pm 2, \ldots).$$

tend to zero as $\varepsilon, \mu, \nu \to 0$. Thus we can say that the realized critical case has infinite dimension.

The main result is that the original problem reduces to the so-called quasinormal form. This is a family of nonlinear equations that don't depend on small parameters whose solutions give the principal parts of the asymptotic approximation of the solutions of the original problem uniform for all $t \geq 0$. Depending on ration between small parameters quasinormal forms may be parabolic equations with one or two spatial variables [2].

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On a wild spiral attractor in four dimension flows

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Until recently, only the hyperbolic attractors and Lorenz attractors could be considered as genuine strange attractors of smooth dynamical systems. However, the situation has been changed after the work of Turaev and Shilnikov [1], where a new class of genuine strange attractors was introduced, the so-called wild hyperbolic attractors. These attractors, unlike hyperbolic and Lorenz ones, admit the existence of homoclinic tangencies. However, these tangencies, unlike homoclinic tangencies in systems with quasiattractors, do not lead to the appearance of stable periodic orbits.

In this work we present an example of a four-dimensional flow with a wild spiral attractor containing an equilibrium state of the saddle-focus type. One of the main features of the Turaev-Shilnikov spiral attractor is that it possesses an pseudo-hyperbolic structure. Speaking shortly, this feature means that, in a neighborhood D of the attractor, there is a "weak" version of hyperbolicity: there is a partition of D into subsets (strongly contracting and volume expanding), that are transversal and invariant with respect to the differential, such that on one of them there takes place exponential contraction along all directions, and on the other exponential expansion of the volume. It is also required that such a partition depends continuously on a point from D.

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Transient and periodic spatiotemporal structures in a reaction-diffusion-mechanics cardiac system

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The reaction-diffusion-mechanics models are the models used to describe self-consistent electromechanical activity in a cardiac muscle. Such models couples two mechanisms of signal spreading in the tissue: the slow (reaction-diffusion) spreading of electrical excitation and the fast (almost instantaneous) spreading of mechanical deformations. This coupling may significantly modify the electrical excitation spreading and corresponding contractile activity with emergence of new spatiotemporal structures and patterns, which modification is not yet completely understood even in the one-dimensional case of a single muscle fiber. We propose clear convenient model which allows one to study the electromechanical activity of such a fiber in relation to the mechanical parameters of fiber fixation (such as stiffness of tissue fixation and the applied mechanical load, which can be easily controlled in experiments). Using this model, we determine and analyze the physical origin of the primary dynamical effects which can be caused by electromechanical coupling and mechanoelectrical feedback in a cardiac tissue.

On the basis of the reaction-diffusion-mechanics model with the self-consistent electromechanical coupling, we have numerically analyzed the emergence of structures and wave propagation in the excitable contractile fiber for various contraction types (isotonic, isometric, and auxotonic) and electromechanical coupling strengths. We have identified two main regimes of excitation spreading along the fiber: (i) the common quasi-steady-state propagation and (ii) the simultaneous ignition of the major fiber part and have obtained the analytical estimate for the boundary between the regimes in the parameter space. The uncommon oscillatory regimes have been found for the FitzHugh—Nagumo-like system: (i) the propagation of the soliton-like waves with the boundary reflections and (ii) the clusterized self-oscillations. The single space-time localized stimulus has been shown to be able to induce long-lasting transient activity as a result of the after-excitation effect when the just excited fiber parts are reexcited due to the electromechanical global coupling. The results obtained demonstrate the wide variety of possible dynamical regimes in the electromechanical activity of the cardiac tissue and the significant role of the mechanical fixation properties (particularly, the contraction type), which role should be taken into consideration in similar studies. In experiments with isolated cardiac fibers and cells, these parameters can be relatively easily controlled, which opens a way to assess electrical and mechanical parameters of the fibers and cells through analysis of dynamical regimes as dependent on fixation stiffness and external force. In real heart, high blood pressure and hindered blood flow play similar role to the applied external force and increased fixation stiffness. Our results provide a hint of how such global (i.e., associated with the large areas of the heart tissue) parameters can affect the heart electrical and contraction activity.

On periodic perturbations of an asymmetric pendulum type equation

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We study time-periodic perturbations of an asymmetric pendulum type equation close to an integrable equation. We find structures of resonance zones and global behavior of solutions of this equation in the cells separated from unperturbed separatrices. The problem of the existence of homoclinic structures in the neighborhood of unperturbed separatrices is discussed. We reveal possible cases of relative position of the separatrices of a trivial fixed saddle point for the Poincare map. The bifurcation diagram for the Poincare map on the plane of the control parameters separating domains of existence of different homoclinic structures is constructed.

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On homoclinic attractors of three-dimensional systems with constant divergency

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In this paper we focus on the problem of existence of homoclinic attractors in three-dimensional flows $\dot{x} = y, \dot{y} = z, \dot{z} = Ax+By+Cz+g(x, y), g(0, 0) = g'_x(0, 0) = g'_y(0, 0) = 0$. Homoclinic attractors are the strange attractors which contain only one (saddle) equilibrium point. The type of such attractors is defined by eigenvalues of the equilibrium point, which depend only on parameters A, B, and C. A method of saddle charts (two-parameter diagram in which regions with different eigenvalues are drawn with different colors) along with methods of charts of maximal Lyapunov exponent and charts of the distance between an attractor and a saddle point (to verify that a saddle point belongs to the attractor) are used for searching and classifying of homoclinic attractors. Using these methods we found attractors of Spiral and Shilnikov types as well as non-symmetrical Lorenz-like attractor.

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Euler characteristic of a surface glued of a $2n$-gon

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The result was obtained in collaboration with G.N. Talanova and O.V. Pochinka.

We consider surfaces glued of $2n$-gon. D. Zagier and J. Harer once concidered such orientable surfaces and described the number of ways of gluing a $2n$-gon to an orientable surface of genus q (the Harer-Zagier numbers).

The canonical variant of gluing the $4q$-gon creating the orientable surface of genus q. But this variant is not single, and one can glue the same polygon to the surfaces of different genus (for example, the square one can glue to the torus, the sphere, the Klein bottle, the projective plane). It is simple to recognize orientability of the received surface. But to calculate the genus or the Euler characteristic is quite hard work for surfaces that may not be orientable.

To solve the problem we triangle the polygon and put in correspondence with it the three-colour graph. Each vertex of the graph is incident to three edges of three colour: u, s or t. Each vertex corresponds to a triangular region of the $2n$-gon, each edge corresponds to common segment of boundary that restrict regions corresponding to the vertices incident to the edge. We mean half-diagonals as t-sides, half-medians as u-sides, half-sides of the $2n$-gon as s-sides.

The main result of our work is that the Euler characteristic $\chi(S)$ of a surface S may be calculated by the formula

$$\chi(S) = \nu - n + 1, \tag{10}$$

where ν is the number of st-cyces of the three-colour graph.

Our proof we base on the Morse-Smale flows theory.

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Local bifurcations in the periodic boundary value problem for one version of the generalized Kuramoto-Sivashinsky equation

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In connection with the problems of describing of the nanorelief the following equation (see, for example, [1])

$$u_t + u_{xxxx} + \beta u_{xx} + \gamma u_{xxx} + a_1(u^2)_{xx} + a_2(u^2)_{xxx} = 0 \tag{1}$$

was proposed ($\beta, \gamma, a_1, a_2 \in R$). Eq. (1) is called "the conserved Kuramoto-Sivashinsky equation with broken parity". Usually, Eq. (1) is considered with the periodic boundary conditions. Without violation of generality, one can assume that

$$u(t, x + 2\pi) = u(t, x). \tag{2}$$

At the analysis of the boundary value problem (1), (2), it is advisable to discriminate two cases: 1) $a_1 = 0$; 2)$a_1 \neq 0$, but always $a_2 \neq 0$. Here, we consider the premier case $a_1 = 0$, but always the right side of Eq. (1) has zero spatial mean. Therefore,

$$M_0(u) = \frac{1}{2\pi}\int_0^{2\pi} u(t,x)dx = C,\ c \in R.$$

Consequently, any solution is representable as $u(t, x) = c + w(t, x), c \in R, M_0(w) = 0$.

For $w(t, x)$ we obtain the following boundary value problem

$$w_t = A(c) + F(w),$$

$$w(t, x + 2\pi) = w(t, x),\ M_0(w) = 0,$$

where $A(c)w = -w_{xxxx} - bw_{xx} - aw_{xxx}$, $b = \beta$, $a = \gamma + 2a_2c\,(a_1 = 0)$, $F(w) = -a_2(w^2)_{xxx}$.

The linear differential operator (LDO) $A(c)$ has the family of eigenvalues $\lambda_n = \tau_n \pm i\sigma_n$, where $\tau_n = -n^4 + bn^2, \sigma = \sigma_n(c) = an^3$ corresponding to the eigenfunction $\exp(inx), n = \pm1, \pm2, \ldots$

Let $b = 1 + \nu\varepsilon, \nu = 16a_2^2/(3(4 + a^2)) > 0$. Consequently, $\lambda_{\pm1}(\varepsilon) = \nu\varepsilon + i\sigma_1, \sigma_1 = a$.

Determine LDO $A(\varepsilon)$ as $A(\varepsilon)v = A_0v + \varepsilon Bv, A_0v = -v_{xxxx} - v_{xx} - av_{xxx}$, $Bv = -\nu v_{xx}, v = v(x)$ and consider the nonlinear boundary value problem

$$w_t = A_0w + \varepsilon Bw + F(w), \tag{3}$$

$$w(t, x + 2\pi) = w(t, x),\ M_0(w) = 0. \tag{4}$$

Theorem 1. There exists $\varepsilon_0 > 0$ such that for all $\varepsilon \in (0, \varepsilon_0)$. Problem (3),(4) has orbitally exponentially stable cycle $Cl(\varepsilon)$ formed by the periodic solution

$$w_p(t, x, \varepsilon, c) = \varepsilon^{1/2}w_1(t, x) + \varepsilon w_2(t, x) + \varepsilon^{3/2}w_3(t, x) + o(\varepsilon^{3/2}),$$
$$w_1(t, x) = q(t, x) + \overline{q}(t, x),\ w_2(t, x) = \eta_2 q^2(t, x) + \overline{\eta}_2\overline{q}^2(t, x),$$
$$w_3(t, x) = \eta_3 q^3(t, x) + \overline{\eta}_3\overline{q}^3(t, x),\ q(t, x) = \exp(ix + i\sigma(\varepsilon, c)t + \varphi_0),\ \varphi_0 \in R,$$
$$\sigma(\varepsilon, c) = a + \varepsilon w + o(\varepsilon),\ \omega = -\frac{8a}{3(4 + a^2)},\ \eta_2 = -\frac{4(a - 2i)}{3(4 + a^2)}a_2,\ \eta_3 = -3\frac{6 - a^2 + 5ia}{a^4 + 13a^2 + 36}a_2^2.$$

Let us return to the boundary value problem (1), (2) with $a_1 = 0, b = \beta = 1 + \nu\varepsilon$. The cycle $Cl(\varepsilon)$ of the boundary value problem (3), (4) generate the local attractor $M_2 = M_2(c, \varphi_0)\,(dimM = 2)$ of the principal boundary value problem (1), (2). For the solutions belonging to M_2 the representation

$$u(t, x, \varepsilon) = c + w_p(t, x, \varepsilon, \varphi_0, c) \tag{5}$$

is valid. The invariant manifold M_2 is filled with periodic solution of the period $T(c) = 2\pi/|\sigma|$, $\sigma = \sigma(c)$ if $\sigma \neq 0$. If $\sigma = 0$ we have the family of equilibrium states.

Theorem 2. All solutions (5) are unstable in the sense of the Lyapunov definition for the norm of the Sobolev space $W_2^4[0, 2\pi]$.

Here, the part of the results from the paper [2] is presented where the case $a_1 \neq 0$ was also studied.

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Rational parametrisation approach in some dynamical systems problems

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In this talk we will show how the use of rational parametrisations can ease the study of the number of solutions of many systems which involve polynomial equations and square roots of some polynomials. We illustrate its effectiveness applying it to several problems appearing in dynamical systems. Our examples include Abelian integrals, Melnikov functions and a couple of questions in Celestial Mechanics: the computation of some relative equilibria and the study of some central configurations.

This is a joint work with Armengol Gasull and Joan Torregrosa from Universitat Autònoma de Barcelona.

On interrelations of divergence-free and Hamiltonian dynamics

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It is a rather frequent case when the study of liquid flows in the Lagrangian description discovers structures characteristic for Hamiltonian dynamics. To show the interrelations between these two types of dynamics we present some calculations which have to demonstrate how symplectic 2-dimensional maps arise in the Lagrangian description of liquid flows.

The investigations carried out show that if the flow generated by a divergence-free vector field has a cross-section, then the related Poincaré map is symplectic and all known results on such maps are applicable. But, of course, it is not obligatory, when the flow has a global cross-section. For instance, the well known ABC flow most likely does not have global cross-section. On the other hand, majority of such flows have periodic orbits, hence local cross-sections exist.

If a divergence-free vector field X on an oriented smooth 3-manifold (M, Ω) have a global cross-section, then M is diffeomorphic to the suspension over a diffeomorphism $P : N \to N$ and a roof function $F : N \to \mathbf{R}$ being the return time $F(x)$ for the orbit through $x \in N$.

The constructions made imply that there is a symplectic 2-form ω on N such that P is a symplectic diffeomorphism w.r.t. $\omega : P^*\omega = \omega$. We show that there is a 4-dimensional smooth symplectic manifold $(\tilde{M}, \Lambda)$ and a smooth Hamilton function H on $\tilde{M}$ such that the Hamiltonian vector field X_H on some its level $H = c$ coincides with the vector field X.

Now suppose a vector field on M is *integrable*, that is, it has a smooth integral F which satisfies the identity $dF(X) \equiv 0$. Then M is foliated into levels of this function $F = c$. A natural question in this case arises: do some restrictions exist on the topology of levels of function F and flows generated by X on the invariant subset $F = c$. Recall that usually the integrability of 3-dimensional vector fields requires to have two independent (almost everywhere) integrals.

Proposition. If X has a discrete set of equilibria, then almost all nondegenerate compact levels of F are 2-tori. The flow on such a torus has not equilibria and preserves a smooth measure.

Let us choose some angle variables (φ, ψ) on Σ. Then 2-form ω_n takes the form $a(\varphi, \psi) d\varphi \wedge d\psi$ with the smooth doubly periodic positive a and the vector field has the form $\dot{\varphi} = A(\varphi, \psi)$, $\dot{\psi} = B(\varphi, \psi)$, where $A^2 + B^2 \neq 0$ and both smooth functions A, B are doubly periodic. Measure preservation means the identity holds $\frac{\partial}{\partial \varphi}(aA) + \frac{\partial}{\partial \psi}(aB) = 0$. Denote

$$\lambda_1 = \int\limits_{\Sigma} A a d\varphi \wedge d\psi, \ \lambda_2 = \int\limits_{\Sigma} B a d\varphi \wedge d\psi.$$

The main role in the orbit dynamics on the torus Σ plays the number $\lambda = \lambda_1/\lambda_2$ called the Poincaré rotation number. As is known, if λ is rational or one of λ_i is equal to zero, then all orbits of the flow are periodic (this is because of the existence of a smooth invariant measure). But if λ is irrational and the flow is of smoothness C^2 then all orbits on the torus are transitive. More subtle effects of ergodicity of the flow are related with the arithmetic type of λ and a smoothness of functions A, B (Kolmogorov).

The integrability of a Hamiltonian vector field X_H on some its level does not imply its integrability on the whole phase space. As an example, consider a perturbation of an integrable vector field $H = H_0 + \varepsilon H_1$. One can choose the function H_1 in the form $H_1 = (H_0 - c)F$ where c is a fixed constant such that on the level $H_0 = c$ the integrable system X_{H_0} has some integrable structure, and a function F can be taken arbitrarily. Let $J_x : T_x^* M \to T_x M$ be the isomorphism between 1-forms and vector fields on M defined by the symplectic form Λ. Then JdH is the Hamiltonian vector field generated by function H. Thus we get

$$J(dH_0 + \varepsilon F dH_0 + \varepsilon(H_0 - c)dF).$$

On the set $H = c$ we have $(H_0 - c)(1 + \varepsilon F) = 0$, thus it is a level of the function H and therefore is the invariant submanifold where the dynamics is integrable since on this level $X_H = J(1 + \varepsilon F)dH_0$ that is obtained by the change time from the integrable vector field X_{H_0} on the level $H_0 = c$ and so has the integrable structure. It is evident that function F can be chosen in such a way that the complete dynamics would be nonintegrable.

About Bifurcations in a Parabolic Equation with Small Diffusion and Deflection Space Variable

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We consider bifurcations of self-oscillations in a boundary-value problem of parabolic type with small diffusion and deviation of the spatial variable.

$$\frac{\partial u}{\partial t} = \varepsilon \frac{\partial^2 u}{\partial x^2} + \kappa(u_{x+s}) + F(u), \qquad 0 < \varepsilon \ll 1$$

with periodic boundary conditions

$$u(t, x + 2\pi) \equiv u(t, x), \qquad (x \in (-\infty, +\infty)).$$

It is shown that the critical case in the problem of a stable point stability has infinite dimension. On the basis of the method of normal forms, it is possible to obtain certain universal systems of equations of parabolic type. The established regimes of these equations make it possible to determine the structure of the solutions of the original boundary value problem. A characteristic feature of these solutions is strong oscillation over the spatial variable, and situations are also possible where, as the parameter before the diffusion coefficient decreases, an infinite change in the "birth" and "death" of a stable autowave occurs. The simplest critical cases are considered in a certain sense. An algorithm for the asymptotic investigation of the entire set of solutions lying in a neighborhood of the equilibrium state is developed. To describe them on the basis of the method of normal and quasinormal forms, it is shown that the dynamic properties of solutions have a high sensitivity to a change in the "small" parameter characterizing the diffusion coefficient.

The gains and losses of synchrony in multiplex neural-glial networks

S. Makovkin, M. Ivanchenko, S. Jalan, A. Zaikin

In work we investigate impact of the glial cells activities on synchronizability of neural cells in multiplex networks framework. Connections among the glial cells form a regular star like periodical structure in which each cell is connected to the four other neighbour cells whereas connections, among glial cells are represented by an Erdős - Rényi random network with average quantity connections is equal by four.

The dynamical evolution of the nodes in this multiplex network is given by bidirectionally coupled phase oscillators:

$$\frac{d\theta_i}{dt} = \omega_i + \sum_{j=1}^{N\times N\times 2} \sigma_{ij} A_{ij} sin(\theta_j - \theta_i) \qquad (11)$$

where θ_i, ω_i are phase and natural frequency of the i_{th} oscillator, the latter taken randomly from uniform distributions. Characteristic time scales of neural spiking are about an order of magnitude faster than the time scale of chemical dynamics of glial cells, hence, we set the mean frequencies $\omega_0^{(g)} = 1$ and $\omega_0^{(n)} = 10$. $A_{ij} = 0, 1$ are adjacency matrix elements, and coupling strength takes values $\sigma_{i,j} = \sigma_g, \sigma_n, \sigma_{ng}$, specific for interglial, interneural, and glial-neural interactions, respectively.

At first case we focus on the case when neural and glial layers are not coupled. Our aim is twofold: we want to capture the effect of network topology on synchronization and study size dependence. The main results in case of uncoupled layers are here:

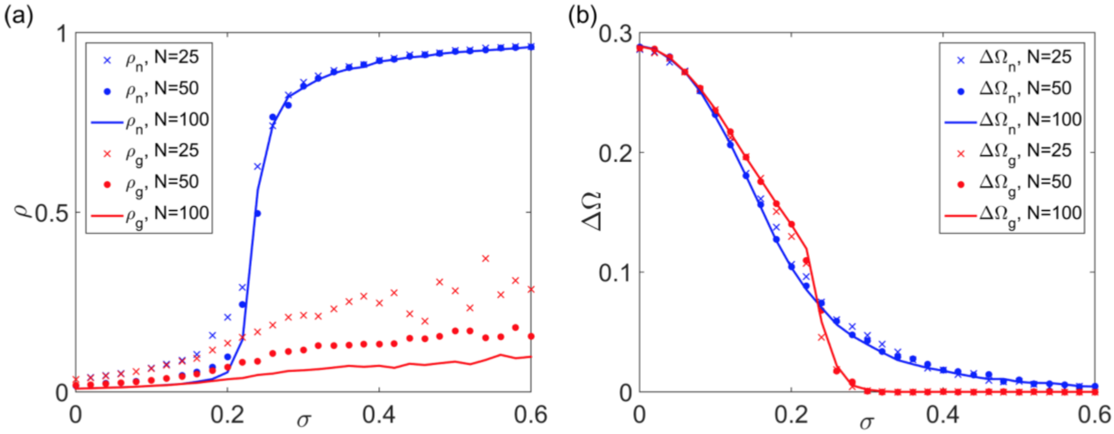

Figure 2: (Color online) Kuramoto mean-field (a) and frequency (b) synchronization in the layers of different topology in dependence on the system size, $N \times N$. Here $\sigma_n = \sigma_g = \sigma$ and the interlayer coupling is zero, $\sigma_{ng} = 0$.

- Kuramoto order parameter r in neural layer does not depend from layer size and has classical Kuramoto like behaviour ("all-to-all" links).
- In glial layer Kuramoto order parameter strongly depends from layer size: r_{glial} decrease due to layer size increasing .
- In limit $N \to \infty$ parameter $r_{glial} \to 0$, that correspond to 1-D nodes chains (they has no mean field) .

At second case neural and glial layers are coupled. We can conclude several points about synchronization case:

- Mean field in glial layer is born with the interaction of neural layer.
- There is partial desynchronization in glial and neuron layers.
- There is abrupt transition to synchronization.

The work is supported by the RSF (Agreement No. 16-12-00077).

Omega-limit sets of C^1 Anosov diffeomorphisms

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We discuss some interesting properties of Milnor attractor and SRB-measure for C^1 Anosov diffeomorphisms. Sometimes C^1 Anosov diffeomorphisms have the non-trivial Milnor attractor and SRB-measure (this is not the case for $C^{1+\alpha}$). We now know that the support of SRB-measure is exactly a Smale horseshoe of zero measure in our exapmle. Still, typically the Milnor attractor of a C^1 Anosov diffeomorphism is the whole phase space. Actually, the set of Anosov diffeomrphisms with non-trivial attractors avoiding the open set is nowhere dense.

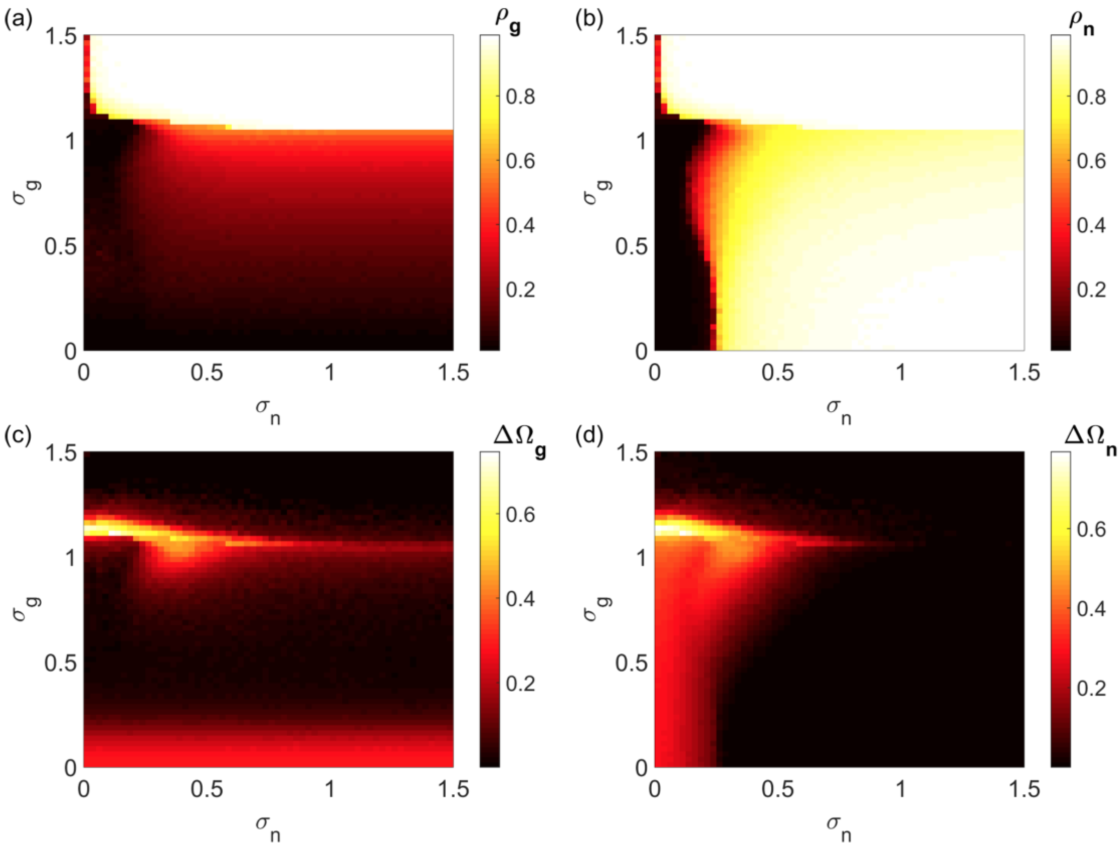

Figure 3: (Color online) Two-parameter diagrams for the color-coded subnetwork order parameters, ρ_g and ρ_n (a, b), and frequency standard deviations, $\Delta\Omega_g$ and $\Delta\Omega_n$ (c, d). Here $N \times N = 100 \times 100$.

Our results also demonstrate one have no hope for zero Hausdorff dimension of the SRB-measure support or Milnor attractor for any Anosov diffeomorphism or circle doubling. On the other hand, we can construct the explicit example of a topological doubling with only one point SRB-measure support, but this is correct neither for a smooth doubling, nor for Milnor attractors.

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On quasi-periodic perturbations of two-dimensional Hamiltonian systems

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We study quasi-periodic (with m frequencies) non-conservative perturbations of two-dimensional Hamiltonian systems. The matter of solutions behavior in the neighborhood of resonance and non-resonance levels of energy is discussed. We find conditions for the existence of resonance quasi-periodic solutions (m-dimensional resonance tori). Our results are illustrated by an example of the Duffing equation.

Entire solutions with various periodic structures for some semi-linear elliptic equations

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We construct entire bounded solutions to the equation

$$\Delta u - u + u^3 = 0 \qquad \text{in} \quad \mathbb{R}^2$$

which have various types of symmetries (rectangular, triangular and hexagonal). Our approach allows us to construct them using concentration theorems and symmetry considerations. We also discuss some generalisations.

The wild Fox-Artin arc in invariant sets of dynamical systems

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In the present paper we show the role that the wild arc of Fox-Artin [1] plays in dynamics. The Fox-Artin arc appeared in dynamics For the first time in 1977 thanks to D. Pixton [2]. Pixton constructed a Morse-Smale diffeomorphism on the 3-sphere with a unique saddle point such that one of the unstable separatrices of this saddle and its stable separatrix form the Fox–Artin arc. This "wild" property showed that an energy Morse function does not exist for Morse-Smale diffeomorphisms in general (notice that for arbitrary Morse-Smale flows an energy function always exists). In 2005 K. Kuperberg [3] constructed on any arbitrary orientable 3-manifold without boundary a continuous flow with a discrete set of fixed points and such that the closure of every non-trivial semi-trajectory is the Fox–Artin arc.

In the present paper we show how the Fox–Artin arc naturally emerges as an element of a heteroclinic intersection for regular 4-diffeomorphisms.

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Nonlinear Dynamics in Two Models of Bubble Contrast Agents

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In this talk we consider dynamical systems for the description of the oscillations of spherical gas bubbles in a liquid. We investigate two models, in one of which a bubble is encapsulated in a shell, and in another one we considered a non–shell bubble. In both cases bubbles are oscillating close to an elastic wall of finite thickness under the influence of external pressure field. The shell term is chosen according to the de-Jong model. These models can be used for the simulation of the behavior of an ultrasound contrast agent close to a blood vessel wall. The main aim of this work is to investigate nonlinear dynamics of spherical gas bubbles within the above mentioned models and determine possible types of attractors existing in these systems.

We show that for the case of a non-encapsulated bubble there can exist a single periodic attractor, multiple periodic attractors, a single chaotic attractor and a chaotic attractor coexisting with periodic one. The last scenario seems to be the most complex one for this model. Note that the considered dynamical systems do not have any fixed points, thus, we use the perpetual points method for localization of the regions where attractors can coexist. We show that this method does not always indicates areas of coexisting of attractors. We demonstrate that there are examples, where perpetual points do not exist in a reasonable subset of the initial conditions space, but different attractors can coexist. A similar situation can be observed in the case of existence of two different attractors and only one perpetual point.

As far as the de-Jong model for an encapsulated bubble is concerned, it seems that there are no areas of physically realistic parameters where two different attractors coexist. On the other hand, we demonstrate existence of a single either periodic or chaotic attractor for different values of the parameters in this system. We also investigate the dependence of type of dynamics on the two control parameters that are magnitude and frequency of the external pressure field. Consequently, we conclude that shelled bubbles much less tend to unpredictable behavior, to which coexistence of attractors of different types can lead.

This work is supported by Russian Science Foundation, grant number 17-71-10241.

On influence of periodical perturbation on homoclinic trajectory to saddle-focus equilibrium in Shilnikov attractors

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In this work the influence of periodical perturbation on homoclinic trajectory to saddle-focus equilibrium in Shilnikov attractors is studied. It is well known that homoclinic loop to saddle-focus equilibrium in Shilnikov attractors for flows is nontransversal. In the same time for discrete Shilnikov attractors stable and unstable invariant manifolds intersect transversally and this homoclinic structure does not disappear with small changes in parameters. Dependence of size of existence interval such homoclinic structure on value of perturbance is studied.

Morse-Smale dynamical systems with few non-wandering points

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We consider Morse-Smale flows and diffeomorphisms with the non-wandering set consisting of three fixed points (two nodes and a saddle). We study the topological structure of supporting manifolds. The question of the topological classification is considered also.

This work was supported by Russian Scientific Found (RNF), project 17-11-01041.

Generator of fractal voltage: practical scheme

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In the report presented the electrical scheme of the device which is able to give out signal:

$$W_M(t) = \sum_{n=1}^{M} a^n \cdot \cos(b^n \cdot \omega_0 \cdot t + \varphi_n), \tag{1}$$

with parameters $0 < a < 1$, $b > 1$ and $a \cdot b > 1$ is under consideration. In formula (1) phases φ_n are independent random values uniformly distributed on the interval $[0, 2 \cdot \pi]$.

It is easy to check that under quite large M function (1) uniformly approximates the well-known Weierstrass function with the same parameters:

$$W(t) = \sum_{n=1}^{\infty} a^n \cdot \cos(b^n \cdot \omega_0 \cdot t + \varphi_n). \tag{2}$$

The autocorrelation function of the signal (1) is equal to:

$$< W_M(t) \cdot W_M(t+\tau) >= \frac{1}{2} \cdot \sum_{n=1}^{M} a^{2n} \cdot \cos(b^n \cdot \omega_0 \cdot \tau), \tag{3}$$

therefore it follows from expression (3) that when the output signal (1) with $b > 1/a^2$ is passed through the correlator we again obtain uniform approximation of the Weierstrass function.

The device considered is the practical example for the A.A. Potapov's concept of fractal radio systems and devices [1]. On the other hand the device suggested one can use in analogous simulation of growth of fractal solid state surface with cylindrical generatrix [2].

The authors prepared this article during working on the project "Leading Talents of Guangdong Province", № 00201502 (2016-2020) in the JiNan University (China, Guangzhou).

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Modelling of the Rikitake system by means of methods of quantum physics

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In the report we apply a new method for investigation of transition to chaos in the Rikitake system [1]:

$$\begin{cases} \dot{x}_1 = -\mu_1 \cdot x_1 + x_3 \cdot x_2 \\ \dot{x}_2 = -\mu_1 \cdot x_2 + (x_3 - \mu_2) \cdot x_1 \\ \dot{x}_3 = 1 - x_1 \cdot x_2 \end{cases}, \qquad (1)$$

namely according to a general approach to modelling of classical dynamical systems through their complement to quantum states presented in [2] let us consider the following system for conjugate momenta of the Rikitake system:

$$\begin{cases} \dot{y}_1 = \mu_1 \cdot y_1 + (\mu_2 - x_3) \cdot y_2 + x_2 \cdot y_3 \\ \dot{y}_2 = -x_3 \cdot y_1 + \mu_1 \cdot y_2 + x_1 \cdot y_3 \\ \dot{y}_3 = -x_2 \cdot y_1 - x_1 \cdot y_2 \end{cases}. \qquad (2)$$

In order to develop results obtained in [3] in this report we estimate both bispectra of system (2) and mutual bispectra of systems (1) and (2).

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On asymptotic description for wave of charge in homogeneous circuit with inversely shifted p-n junctions

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Let us consider a homogeneous chain consisting of a large number of identical four-ports. Futhermore let each of them contains resistance, inversely shifted p-n junction connected in series with the resistance and capacitor. Under small characteristic size of elementary cell of this chain one can describe this circuit by means of nonlinear parabolic equation for electrical charge on p-n junction. And one can find out solution of this equation as travelling wave. If velocity of such wave is quite small then in the framework of the theory of quasi-Hamiltonian systems [1] it is possible to construct explicit asymptotic solution of the equation for the shape of this wave.

The report presented illustrates general approach for asymptotical analysis of travelling waves in reaction-diffusion equations which are essential for biological and ecological applications [2]. Possibility of investigation of this kind of equations by means of SPICE modelling [3] is also discussed.

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On the Kulbak-Leubler entropy in quantum mechanical systems

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Using the well-known statistical interpretation of wave function in this report we extend definition of the Kulbak-Leibler entropy from classical stochastic systems [1] to nonrelativistic quantum mechanical systems. To show the constructivity of such transfer in the report presented we calculate directly the Kulbak-Leibler entropy for a number of one-dimensional quantum mechanical examples namely for free moving particle, for particle in homogeneous field and for harmonic oscillator.

This report continues the line started earlier (see [2] and references there in) and directed on search of analogies between radiophysics and quantum mechanics. Applications of the Kulbak-Leibler entropy for solutions of the nonlinear Schrödinger equation and its generalizations [3] are also discussed.

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Some notes about dynamical chaos on the smallest scales of the Universe

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This review continues the line started at report [1] demonstrating interrelations between the theory of non-abelian gauge fields and theory of dynamical systems.

The main topic of the report presented is to describe the chaotic behaviour of an anisotropic universe in the vicinity of cosmological singularity in accordance with papers [2, 3]. Antropogenic principle which selects the Universe with actually observed characteristics [4] and gives us the possibility to investigate neural networks dynamics [5] is also under consideration.

The purpose of this survey is to attract attention of young representatives of L.P. Shilnikov's school for nonlinear dynamics to research work in the sphere of nonlinear theory of fundamental fields.

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Total harmonic distorsions for voltage and for superconducting current in Josephson junction

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The technique developed in the report [1] for exact calculation of total harmonic distorsions (THD) for voltage in oscillatory circuit with ferroelectric capacitor with negative differential capacitance proves to be very fruitful.

In the report presented the technique suggested is applied to another nonlinear dynamical system namely to ideal Josephson junction. This device is described by differential equation of simple pendulum:

$$\varphi''+\sin\varphi=0, \qquad (1)$$

where φ is equal to difference of phases of wave functions of superconductors forming the Josephson junction.

Starting from the well-known exact solutions of equation (1) we have found THD for voltage on Josephson junction $V=\varphi'$ and THD for superconducting current $I=\sin\varphi$ via Josephson junction.

We underline that our calculations have been done for both oscillating and rotating regimes of equation (1).

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Waves in Large Disordered Anisotropic Fractal Systems, in Clusters of Small-Size Space Vehicles, in Synthesized Space Antenna Aggregations - Cluster Apertures, and in Radar

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The up-to-date status of researches in the problems of multiple scattering of waves by fractal discrete random media is presented. The multiple scattering theory for waves in media containing random scatterers has been learned by many authors [1-7]. In case of statistical consideration of waves scattering they base on the stochastic wave equation or on a system of such equations which the problem of wave's diffraction on a statistical ensemble is investigated and formulated for. Although all these researches led to discovery of some basic physical principles there are still many problems concerned the multiple scattering in fractal media. Issues of the general theory of multiple scattering of electromagnetic waves in fractal random media on the basis of the Foldy-Tversky classical theory modifications are considered in detail [1-3]. A designed modification of the non-single scattering theory allowed including values of fractal dimension D and fractal signature $D(\mathbf{r}, t)$ of disordered large system into consideration The radar equation has been analytically considered for an extremely fractal medium. Theoretical researches are agreed with previously published results of foreign authors [4, 7]. Similarly, one can prove the solution for anisotropic irregular fractal systems: fractal cascades enclosed to each other, graphs of fractal chains, percolation systems, space rubbish, clusters of drones or small-size space vehicles (SSV) including mini- and micro- classes, dynamical synthesized space antenna aggregations (cluster apertures), space-distributed cosmic systems (clusters) from small SSV for solving problems of emergency monitoring and so on. This research continues the author's series of papers on justification of application of the fractal theory, physical scaling and fractional operators in issues of radio physics and radiolocation [5, 8].

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INDICATORS OF THE KINEMATICS OF THE MOVEMENT OF AQUATIC ORGANISMS, DEPENDING ON THE ANGLE OF INCIDENCE OF MICROWAVE RADIATION OF LOW INTENSITY. THE DEVELOPMENT OF A PROGRAMME OF ENVIRONMENTAL MONITORING.

Indicators of the kinematics of the movement of aquatic organisms, depending on the angle of incidence of microwave radiation of low intensity. The development of a programme of environmental monitoring.

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Environmental management is currently a priority in scientific, technical and economic development of many countries. The most valuable resource of our country is the presence of huge reserves of fresh water in natural water bodies - lakes and rivers. Therefore, the use of innovative technology in the protection of water resources must solve the problems of the status of aquatic biological resources, conducting environmental monitoring on a qualitatively higher level.

Electromagnetic radiation from space is an essential factor under the action of which was the development of the Ecosystems of the Earth and their adaptation. Pronounced biological effect among the entire spectrum of electromagnetic radiation has a microwave range. The mechanism of action of non-thermal microwave radiation on living systems remains poorly understood. However, we know that it is for informational astro research character, which depends on the water content in the tissues of the living object and the angle of incidence of radiation.

Photo.1. The generator of electromagnetic radiation of low intense in the centimetres range «Biorhythm-1»

9 positions were chosen in our experiments with different distances from source of microwave radiation with the angles of incidence. We chose aquarium snails (Planordis corneus var. Rubra) as the studied objects.
In each position, we have investigated the movement of at least 10 individual animals. Processing of snails with microwave radiation power of 0.01 mW and a frequency of 2450 MHz produced by the generator of low intense in the centimetres range,” Biorhythm-1”.We were studying the behaviour of animals and recording the coordinates of their movement. The studied parameters were: the length and the character of the trajectory, the average and the module speed, the orientation of the coordinate axes.

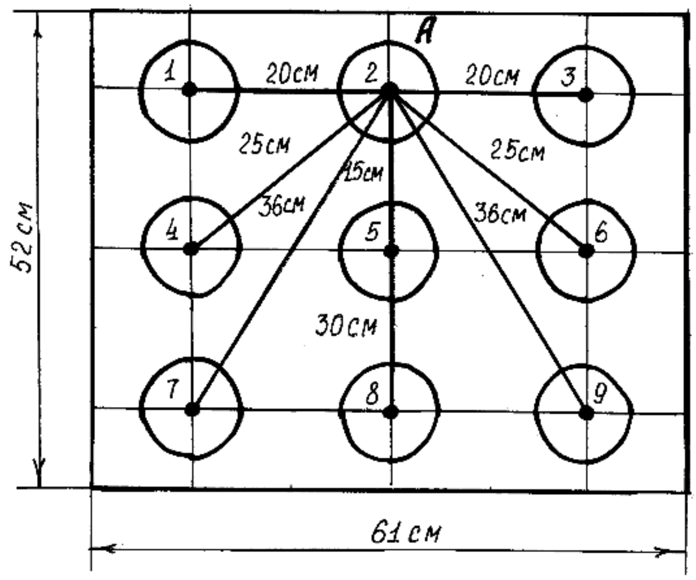
By measuring the coordinates of the movement of snails were required to make calculations of speed and the distance snails made in each minute. Because of the huge amount of coordinate of movement, one of the problems was the time spent on counting the position of a snail. The solution to this problem was a writing a program to calculate the object speed and distance passed for every 60 seconds.

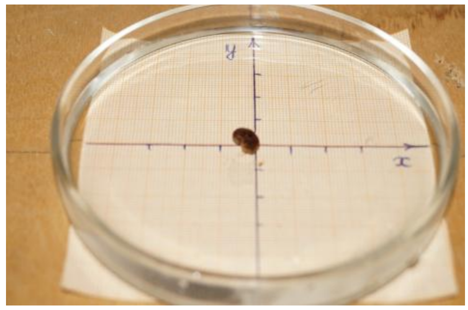
The program was written in C++. One of the advantages of this programming language is a high-performance memory, so it is well suited for this task.

Briefly, the program structure can be described as follows.

The data goes through a text document, in which start points and end points of position with an initial velocity on each coordinate are pre-recorded. Then comes the selection of a specific formula of acceleration and ultimate speed on the abscissa and the ordinate.

Photo.2 The scheme of experiments to study behavioral responses of snails.





We took the snail as a material point. Because the movement of snails was straightforward, there were only three special cases for traffic on each of the coordinates:

If the snail is moving with an initial velocity.

$$a = \frac{2(S - V_0 t)}{t^2}$$

$$V = V_0 + at$$

$$V = V_0 + \frac{2(S - V_0 t)}{t}$$

If the snail is moving, but the initial speed is missing.

$$\begin{cases} a = \dfrac{2S}{t} \\ V = at \end{cases}$$

$$V = 2S$$

If the snail didn't make the distance.

$$a = 0$$

$$V = 0$$

In accordance with the completed data, the program determines each case and considers according to the formulas, then outputs to a text document.

One of the advantages of this program is its performance. The script expects the data in less than a second. But the main problem is that it does not have its own interface and less functional. It cannot be used for another movement because the conditions are not able to change by the user. In the future, it is planned to refine this program for a more accurate description of the motion, determine the location (coordinates) of the object being studied, prediction of the behavior of a biological object, depending on the nature, acting on it radiation.