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BOOK of ABSTRACTS

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Chaotic attractors of certain dynamical systems

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In this talk several concrete dynamical systems having chaotic attractors are presented. Particularly, we exhibit and discuss singularly hyperbolic attractor for a map, quasi-strange attractor for a nonautonomous system and wild attractor for a family of coupled nonlinear oscillators. This work is supported by the Ministry of Education and Science of the Russian Federation (agreement № 02.B.49.21.0003).

Stabilization of cubic nonlinear equation's periodic solution by delayed feedback control

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We researched model equation with qubic nonlinearity that has following form

$$\dot{z} = \sigma z + \gamma |z|^2 z.$$

Here $z(t)$ is complex-valued function; σ and γ are complex parameters.

In assumption, that given equation has unstable periodic solution, we researched the issue of its stabilization by delayed feedback control. Two types of control have been considered: with one and with two delays. They have forms $K(z(t+T) - z(t))$ and $K(z(t+T_1) - z(t)) + K(z(t+T_2) - z(t))$ respectively, where K is complex coefficient of delayed feedback control and $T, T_1, T_2 > 0$ are time delays chosen in such way to ensure the existance of the initial periodic regime in a new system.

It has been shown that for some values of initial system parameters stabilization is possible. For each type of delayed feedback control sufficient and necessary conditions of stabilization have been analytically found. Also we found borders of control parameters area which allow us to turn unstable cycle into stable one. Furthermore, it was demonstrated that usage of the control with two delays gives considerable improvement and allows to reduce area of initial system parameters for which stabilization is impossible.

Synchronization in ensembles of pulse coupled integrate-and-fire oscillators

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Synchronization is a central mechanism for neural information processes that connects different parts of the brain. One of the most general and relevant dynamical phenomenon observed in the mammalian brain is the rhythmic coherent behavior involving different neuronal populations. Many models of pulse-coupled ensembles oscillators have regions in the parameter space where the ensemble evolves neither to asynchronous state nor to fully-synchronized one. These partially-synchronized states can be caused by external influence on the ensemble, however, many ensembles oscillators may evolve to partially-synchronized state even there is no external influence. In this report the model of coupled ensembles oscillators is examined. Each ensemble evolves to partially-synchronized state for certain parameter values. Oscillators in the ensemble are pulse-coupled through a common field on the "any-to-any connection"principle. At first we analyze the dynamics of single ensemble oscillators. Depending on the parameters value, the dynamics is characterized as incoherent, weakly coherent or strongly coherent, based on the nature of the common field dynamics. During the examination of the dynamics of the coupled neural ensembles having different types we investigate evolution modes of the common field and elements of ensembles for various values of the coupling parameter. The mechanisms of transition to frequency synchronization modes, generation and damping of the oscillations in the common field of the ensemble, partial and clustered synchronization of neurons in the ensemble are presented. In our analysis we use analytical and numerical results.

Wave patterns in a neural network with competing regular and irregular couplings

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Many real network systems from various fields of modern science have complex enough, irregular architecture of interelement couplings. There exists a large number of studies on collective dynamics of such systems which are devoted mostly to processes of partial (formation of clusters coordinated oscillatory activity) and full synchronization. But how the irregularity affect the wave processes is not clear enough.

We present some results on the study of spatio-temporal dynamics of a ring of electrically coupled oscillatory Morris-Lecar neurons with extra irregular set of (excitatory or inhibitory) chemical couplings. When there are only the regular couplings the system produces so called "anti-phase wave patterns" looking like the envelope solitons. They have the form of spatiotemporal oscillations with a smooth localized envelope which propagates along the system preserving its shape and velocity. We investigate how this activity patterns evolve when different kinds of irregular couplings are added. To classify collective regimes emerging in irregular networks, in addition to Kuramoto order parameter, we introduce a wave order parameter. It indicates how close a regime to wave one. It is found that when a random set of excitatory couplings are added the wave activity preserves only for small values of their distributing probability and strength. A completely different laws are found when the irregular couplings are inhibitory. Here the wave activity is observed for any value of couplings strength, but small values of distributing probability.

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High-frequency electromagnetic oscillations in excitable tissue .

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Along with use of the initial differential equations of Hodzhkin-Haxley of the fourth order (and their modifications), the equations of Ziman, Alonso-Panfilov of the third order for the description and operational analysis excitable tissue (ET), etc. are used the equations of the Van-der-Pol-duffing (VDPD) the second order (Fitz-Hue, 1961).

Systems of VDPD are applied as a matter of convenience the analysis of locomotion of variables (in particular, currents and strains in ET) and field good resemblance under the form. To voltage, porosity with really measured parametres of impulses in ET.

However, for example, in basic work of Fitz-Hue are not specified time rate in behaviour of variables at generation of self-oscillations, and frequency characteristics of the conforming model of phylum VDPD were not considered.

In this connection, consideration of a range of vibration frequencies in system VDPD in a self-oscillatory mode for an assessment of adequacy of model of phylum VDPD to real locomotions of variables in ET is obviously important.

For such analysis of systems VDPD it is convenient to pass to an equivalent circuit of phylum of the generator on the tunnel diode (GTD), as locomotions of currents and strains completely are described the similar equations.

Dependence of frequency period (in parametrical space of stability), for example, from heatlosses where there is a frequency period upper bound was considered analytically that corresponds 1 ms, or 1 khz.

It is shown, at work GTD (with real biophysical values parameters) in space of steady self-oscillations frequency cannot be more low 1 khz and is in a range (1 - 10000) khz.

It means that at work ET can provoke high-frequency (HF) electromagnetic field which certain impact on functioning alive electicity excitable and unexcitable tissue can make. Degree of such influence can be significant, since voltage of the conforming fluctuations 1 mv, i.e. order amplitude electrogrammes of excitable frames of a tissue.

Lenses with distributed focus.

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Currently used laser cutters create a beam focused to a point and cut a wedge [1]. Thus it is necessary to maintain strictly distance from a lens to the cutting material. There are also laser scalpels, focusing the beam to a point [2]. In order to that the laser didn't cut and burns down further or closer than some distance, focusing of a beam needs to be made distributed. Lets function f(x) is distribution of power on axis OX.

There are described three types of lens, which can to focus laser beam to the segment, and not point – analog of the Fresnel lens, optical inhomogeneous plate and aspherical lens.

As result became equations, describing angle between surface of analog of Fresnel lens, refraction coefficient for inhomogeneous plate and form of aspherical lens.

There are trends for use these lenses in laser surgery and cutting. There is possibility to reduce liquid mass of metal during cutting, if focal distance decreases with radius of elementary ring on lens.

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Bifurcations in a finite dynamical systems

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Consider the dynamical system $f : \mathbb{Z}_{p^n} \to \mathbb{Z}_{p^n}$, where $\mathbb{Z}_{p^n}$ is the ring of residues modulo p^n, p is a prime integer, and f defined as

$$f(x) = \frac{(1+p)^x - 1}{p} \mod p^n.$$

One can check that this function is bijective, so Z_{p^n} is the union of periodic trajectories (cycles) of the form $a_1,\ a_2 = f(a_1),\ \ a_k = f(a_{k-1}), a_1 = f(a_k)$. A cycle of the length k I will call a k-cycle.

If $p > 3$ is a prime then all cycles are p^m-cycles, $m = 0, 1, ...$ When one pass from n to $n + 1$

- a p^m-cycle with $m \neq 0$ turns to p^{m+1}-cycle;

- for a fixed point there are two possibilities:
  - it splits to p different fixed points or
  - it turns to a p-cycle.

Well, all it looks like some kind of a bifurcation.

My motivation to study such problems comes from the group theory. Particularly, it comes from the question if the Higman groups $\langle a, b, w \mid b^{-1}ab = a^r,\ b = w^{-1}aw,\ w^4 = 1\rangle$ are sofic. The challenging problem here is to show that some exponent-like maps have a small number of short cycles.

Nonclassical relaxation oscillations in neurodynamics

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We consider a one-dimensional chain of diffusively coupled neurons with Neumann boundary conditions.

$$\varepsilon\dot{u}_j = v_j - g(u_j) + d(u_{j+1} - 2u_j + u_{j-1}), \qquad \dot{v}_j = a - u_j - v_j, \qquad j = 1, \ldots, m, \tag{1}$$

where $u_{m+1} = u_m$, $u_0 = u_1$, and $d = \text{const} > 0$. Assume that every individual neuron is described by system

$$\varepsilon\dot{u} = v - g(u), \qquad \dot{v} = a - u - v, \tag{2}$$

where $0 < \varepsilon \ll 1$, $a = \text{const} > 0$. Let the following conditions take place.

Condition 1. There exists $u = u_* > 0$ such that
$g(0) = 0$, $\quad g'(u) > 0$ for $u \in (-\infty, u_*)$, $\quad g'(u) < 0$ for $u \in (u_*, +\infty)$,
$g'(u_*) = 0$, $\quad g''(u_*) < 0$, $\quad a - u_* - g(u_*) > 0$.

Condition 2. For $u \to +\infty$, we have the asymptotic representation $g(u) = \alpha_0 + \sum\limits_{k=1}^{\infty} \dfrac{\alpha_k}{u^k}$, $\alpha_0 > 0$, which remains valid after differentiating any times with respect to u.

A singularly perturbed system of ordinary differential equations (2) with a fast and a slow variable is a modification of the well known FitzHugh–Nagumo model from neuroscience. The existence and stability of a nonclassical relaxation cycle in the system (2) are studied. The slow component of the cycle is asymptotically close to a discontinuous function, while the fast component is a δ-like function. The following result is true.

Theorem For any fixed $d > 0$ and all sufficiently small $\varepsilon > 0$, the homogeneous cycle of system (1) is exponentially orbitally stable.

We have shown that diffusion chain (1) exhibits nontrivial dynamics. With a suitable choice of parameters in system (1), along with the stable homogeneous cycle, there are a lot of the other attractors (cycles).

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Lorenz-like attractors in a nonholonomic model of a celtic stone.

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We consider a nonholonomic model of celtic stone movement along the plane. As well-known, the movement of celtic stone on the plane is considered still as one of most complicated and very little studied type of rigid body movements. Moreover, it is one of view types of such movements in which chaotic dynamics is possible. The existence of strange attractors in the celtic stone dynamics was recently discovered by A.V.Borisov and I.S.Mamaev [1]. In the paper [2] these results were extended and main bifurcations leading to chaos appearance were studied. In particular, various types of chaotic dynamics were found in the model: a spiral strange attractor, torus-chaos attractors, nearly conservative chaos and even the so-called mixed dynamics [3]. The latter type of chaotic orbit behavior means that the corresponding nonwandering set contains infinitely many coexisting periodic orbits of all possible types: stable, completely unstable, saddle and, due to reversibility of the system, symmetric elliptic periodic orbits. Moreover, for certain types of celtic stone (possessing certain geometrical and physical properties) strange Lorenz-like attractors were found in their nonholonomic models. In this talk we observe the corresponding results related to scenarios of appearance and break-down of such strange attractors.

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On bifurcations of area-preserving maps with quadratic homoclinic tangencies on non-orientable surfaces

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We study bifurcations of non-orientable area-preserving maps with quadratic homoclinic tangencies. We study the case when the maps are given on non-orientable two-dimensional surfaces. We consider one and two parameter general unfoldings and establish results related to the emergence of elliptic periodic orbits.

Discrete model of the dynamics of a multimode laser with periodically driven pumping

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Dynamics of a multimode laser has been studied in papers [1,2] on the base of the $(2N + 2)$-dimensional system of differential equations, where N is a number of longitudinal modes. The model takes into account cross-saturation of modes due to the effect of spatial hole burning, implies the global coupling (each-to-all) for elements (longitudinal modes) and periodic modulation of the pumping rate. Chaotic pulsing, splay states and cluster states have been numerically demonstrated.

In this paper we analyze conditions for synchronous states on the base of the Poincare mapping which we get by integrating the system asymptotically

[3] in the case of the large ratio of the photon decay time in the cavity and the population inversion relaxation time. That is indeed valid for class B lasers, including semiconductor-based, solid-state, and CO_2 lasers. With such a large parameter the system generates short-width spikes.

The obtained discrete $(2N+1)$-dimensional mapping adequately describes certain synchronous spiking in the original system of coupled oscillators. In particular, the fixed point of the mapping corresponds to the splay state – phase-synchronized oscillations of the period NT, where T is the modulation period. One can reestablish also the oscillation characteristics: pulse energy and time shift between the pulses. By computing the mapping we find the basin of such a regular attractor. In the phase space it coexists with other attractors. Phase-synchronized splay states can be stabilized by selecting the initial conditions in the vicinity of the fixed point of the mapping or by injecting a signal to switch on the attractor desired in multistability domains.

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Топологическая классификация структурно устойчивых каскадов с двумерным неблуждающим множеством на 3-многообразиях

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Доклад посвящен изложению результатов, полученных совместно с Е.В. Жужомой, Ю.А. Левченко, В.С. Медведевым и О.В. Починкой.

Пусть M^3 — гладкое трехмерное замкнутое многообразие и G — класс диффеоморфизмов $f : M^3 \to M^3$, определяемый следующими условиями:

i) любой диффеоморфизм $f \in G$ является структурно устойчивым;

ii) неблуждающее множество $NW(f)$ любого диффеоморфизма $f \in G$ имеет топологическую размерность два.

Из условия ii) и спектральной теоремы С. Смейла следует, что $NW(f)$ состоит из конечного объединения непересекающихся замкнутых инвариантных (базисных) множеств размерности два, ограничение диффеоморфизма f на каждое из которых является топологически транзитивным. В силу [1] условие ii) влечет, что каждое базисное множество является либо аттрактором либо репеллером, а в силу [2] оно является либо растягивающимся аттракторам, либо сжимающимся реппеллером, либо двумерной поверхностью, гомеоморфной тору и топологически вложенной в многообразие M^3. Из [3] следует, что если неблуждающее множество структурно устойчивого диффеоморфизма f заданного на M^3, содержит растягивающийся аттрактор (сжимающийся репеллер) то оно не может содержать других нетривиальных базисных множеств и необходимо содержит по крайней мере один сток (источник) и конечное множество (возможно пустое) седловых периодических точек. Кроме того, в [3] установлено, что в этом случае многообразие M^3 диффеоморфно трехмерному тору, и f топологически сопряжен некоторому обобщенному DA-диффеоморфизму. Таким образом, из [2] и [3] следует, что неблуждающее множество любого диффеоморфизма $f \in G$ состоит из конечного числа поверхностей, каждая из которых гомеоморфна двумерному тору. Более того в [4] доказано, что каждый такой тор ручно вложен в M^3, а ограничение некоторой степени дифеоморфизма f на этот тор топологически сопряжено с диффеоморфизмом Аносова.

В настоящем докладе устанавливается, что многообразие M^3 допускает диффеоморфизм из класса G тогда и только тогда, когда M^3 диффеоморфно многообразию, полученному из прямого произведения двумерного тора на отрезок $\mathbb{T}^2 \times [0,1]$ отождествлением точек $(z,1)$ и $(\tau(z),0)$, где τ гомеоморфизм тора, индуцированный матрицей либо тождественной, либо минус тождественной, либо целочисленной унимодулярной и гиперболической. Построен подкласс $\Phi \subset G$ модельных диффеоморфизмов, топологическая классификация которых получена на алгебраическом языке. Установлено, что любой диффеоморфизм $f \in G$ топологически сопряжен некоторому модельному частично гиперболическому и динамически когерентному диффеоморфизму из Φ.

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Топологическая классификация градиентно-подобных потоков при помощи энергетической функции

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В докладе излагаются результаты, полученные совместно с В.З. Гринесом и О.В. Починкой, и опубликованные в [1].

Рассматривается класс $G(M^n)$ потоков Морса-Смейла на замкнутом гладком многообразии M^n размерности $n \geq 2$, таких, что любой поток $f^t \in G(M^n)$ обладает следующими свойствами:

1. неблуждающее множество $\Omega(f^t)$ не содержит периодических траекторий;
2. Индекс Морса $ind(p)$ произвольной точки $p \in \Omega(f^t)$ (равный размерности неустойчивого многообразия точки p) принадлежит множеству $\{0, 1, n-1, n\}$;
3. инвариантные многообразия различных седловых точек из $\Omega(f^t)$ не пересекаются.

Из работы [2] следует, что для любого потока $f^t \in G(M^n)$ существует *самоиндексирующаяся энергетическая функция* — такая функция Морса $\varphi : M^n \to [0, n]$, что:

1. множество критических точек функции φ совпадает с множеством $\Omega(f^t)$;
2. $\varphi(f^t(x)) < \varphi(x)$ для любой точки $x \notin \Omega(f^t)$ и любого $t > 0$;
3. $\varphi(p) = ind(p)$ для любого $p \in \Omega(f^t)$.

Следуя Р. Тому, будем называть функции $\varphi : M^n \to \mathbb{R}$ и $\varphi' : M^n \to \mathbb{R}$ *топологически эквивалентными*, если существуют сохраняющие ориентацию гомеоморфизмы $H : M^n \to M^n$ и $\chi : \mathbb{R} \to \mathbb{R}$ такие, что $\varphi' H = \chi\, \varphi$.

К. Мейер в [3] доказал, что топологически эквивалентные потоки Морса-Смейла имеют эквивалентные самоиндексирующиеся энергетические функции, а в случае потоков из класса $G(M^2)$ верно и обратное утверждение: топологическая эквивалентность самоиндекирующихся энергетических функций потоков из $G(M^2)$ влечет топологическую эквивалентность этих потоков.

Мы показываем, что этот факт, вообще говоря, неверен в случае $n = 3$, и вводим более сильное условие эквивалентности энергетических функций, названное *согласованной эквивалентностью*. Основной результат доклада содержится в следующей теореме.

Теорема. *Потоки $f^t, f'^t \in G(M^n)$, $n \geq 3$, топологически эквивалентны тогда и только тогда, когда их самоиндексирующиеся энергетические функции согласованно эквивалентны.*

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Multistability of synchronization modes in system of two oscillators with adaptive couplings

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We studied the processes of synchronization in system of two phase oscillators with adaptive couplings. Coupling weights evolve dynamically depending on the relative phases between the oscillators. The main operating modes of such system are the synchronization mode, in which the oscillator frequencies are equal while the difference of their phases takes a certain constant value, and the beating mode, in which the difference of phases grows permanently while the mean frequency difference has a certain constant value. It was also shown that system can demonstrate another type of synchronization mode, in which the mean frequencies of oscillators are equal and the difference of their phases varies in certain limits. There is a parameter region where the system exhibits multistable behavior.

Existence and stability of automodel cycles of distributed in space or time dynamical systems

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We study model differential equation

$$\dot{u} = (1 - (1 + ip)|u|^2)u$$

with two different types of distibution.

As distributed in time equation we consider so-called Stuart-Landau equation

$$\dot{u} = (1 - (1 - ic)|u|^2)u + \gamma e^{i\phi}(u(t - T) - u). \tag{3}$$

The main assumption in this case is that delay $T > 0$ is sufficiently large. We study existence of automodel cycles $u = R\exp(i\Lambda t)$ (here R and Λ are real constants, $R > 0$) and their stability in phase space $C[-T, 0]$.

As distributed in time equation we consider Ginzburg-Landau equation with 2π-periodic boundary conditions

$$\begin{gathered} \dot{u} = (1 - (1 + ib)|u|^2)u + \varepsilon^2(1 + id)u'', \\ u(t, x + 2\pi) \equiv u(t, x). \end{gathered} \tag{4}$$

Here the main assumption is that diffusion $\varepsilon > 0$ is sufficiently small. In this case we study existence of automodel cycles $u = R_k \exp(i\Lambda_k t + ikx)$ (here R_k, Λ_k are real constants, $R > 0$, k is integer) and their stability in phase space $C[0, 2\pi]$.

It is proved that in both cases solutions constitute one-parameter family (main part of solutions lie on one-parameter curve). Explicit formulas for solutions and corresponding one-parameter curves are found. Necessary and sufficient conditions of stability of automodel solutions are found analitically. Location of stable regions on these curves is studied. Hypermultistability is proved in both cases.

Reversal and figure-eight attractor in the nonholonomic model of Chaplygin top

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We consider a model of unbalanced ball moving on a rough plane. By an unbalanced ball we call the dynamically asymmetric ball with a displaced center of gravity. The roughness of the plane means that a body moves without slipping. It is well known [1] that such a motion is governed by the system of 6 differential equations in variables ω (angular velocities) and γ (projection of the vertical unit vector to the body frame). These equations admit 2 integrals: energy and geometric one which reduce the problem dimension from 6 to 4. To visualize the dynamics of this system we construct three-dimensional Poincaré map on some cross-section using the Andoyer-Deprit variables [1].

If the gravity center of the ball is displaced along all 3 axis of the body frame, the dynamics of the ball looks as very complex. In this case we found few types of strange attractors such as torus-chaos and figure-eight attractor. The existence of the figure-eight attractor in three dimensional maps was predicted in [2]. As we know, the system under consideration is the first system from applications where an attractor of such the type was found. In this talk we present results related to scenarios of the appearance of figure-eight attractor and its properties. In more details results will appear in [3].

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Normalization of equations with two different by order delays

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Consider the equation with two delays

$$\dot{x} + x = ax(t - T_1) + bx(t - T_2) + f(x.x(t - T_1), x(t - T_2)), \quad T_1 > T_2 > 0,$$

where $f(x, y, z)$ is nonlinear function ($f(0, 0, 0) = 0$). Main assumption is that both T_1 and T_2 are asymptotically large and $T_1 T_2^{-1}$ is large too. Let $T_1 = \varepsilon^{-1}$, where $0 < \varepsilon \ll 1$. Then $T_2 = \varepsilon^{-1}(k_0 + \varepsilon^{\alpha} k_1)$ ($\alpha > 0$). The problem to research is to determine the behavior of solutions in some small (but independed of ε) neighbourhood of zero equilibrium state. The method of investigations is so-called method of quasinormal forms.

We proof that if $|a| + |b| < 1$ then $z = 0$ is stable and if $|a| + |b| > 1$ then zero is unstable. So $|a| + |b| = 1$ is critical case.

In critical case we construct special evolutionary equations (quasinormal forms). Their non-local dynamics determines the local behavior of solutions of the original equations. The particular kind of quasinormal forms is highly depends on parameter α. There are three different situations: (1) $\alpha < 1$, (2) $\alpha = 1$ and (3) $\alpha > 1$. Also, there are important situation when b is small, so we have small multiplier at the term with largest delay.

Slow passage through a saddle node bifurcation of limit cycles in the model of neuron firing

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The dynamic saddle node bifurcation of limit cycles has been investigated in the modified FitzHugh-Nagumo model of neuron firing in which one of the parameters is slowly varying with time. It was shown that the stable large amplitude oscillations in this system occur on the two-dimensional invariant manifold and continue to exist for a finite time even after the passage through the bifurcation point. The delay time of the oscillation disappearance can to be a very large and it is not negligible. The nonlocal oscillation properties of the model and in particular the threshold properties of the two-dimensional invariant surface of the saddle trajectory underlie the delay phenomenon.

The evolution of two-level system in the presence of self-interaction

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Integro-differential equation for the modeling of quantum system's evolution (under the action of electromagnetic field that generated by the system itself) is proposed. We assume that the system radiates as an electric dipole. This equation can be written as

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} = \hat{H}_0\psi + \frac{i\hbar e}{mc}\Bigg(\Bigg(\frac{1}{c}\int \frac{\left[\frac{d(e\vec{r}_0\psi^*(t,\vec{r}_0)\psi(t,\vec{r}_0))}{dt}\right]_{ret}}{|\vec{r}-\vec{r}_0|}dV_0 + \\ + \frac{1}{4\pi c}\int \frac{\left[\frac{d}{dt} grad \int \frac{[div\ (e\vec{r}_1\psi^*(t,\vec{r}_1)\psi(t,\vec{r}_1))]_{ret}}{|\vec{r}_0-\vec{r}_1|}dV_1\right]_{ret}}{|\vec{r}-\vec{r}_0|}dV_0\Bigg)\nabla\Bigg)\psi. \end{aligned} \quad (1)$$

$\hat{H}_0$ is the Hamilton operation. If the only item with this operator remained in the right member of the equation, we would call it "Schrödinger equation". The index "ret" to the right of square brackets means the retardation [2].

If to look for the approximate solution with the presence of two energy levels, we can receive the system of nonlinear equations for the complex amplitudes of the wave functions like this:

$$\begin{cases} \frac{dC_1}{dt} = \gamma\left|C_2\right|^2 C_1, \\ \frac{dC_2}{dt} = -\gamma\left|C_1\right|^2 C_2, \end{cases} \qquad \gamma = \frac{2e^2}{3c^3\hbar}\omega_{21}^3\left|\vec{r}_{21}\right|^2. \quad (2)$$

Value γ is the Einstein coefficient of spontaneous emission divided on two.

The solution of this system describes the spontaneous transition from the high energy level to the low. The electric current corresponding to this process is the high-frequency occilation (with the frequency of quantum transition). It is modulated by bell-shaped function equal the product of modules of amplitudes (C_1 and C_2).

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$m : n$ synchronization of oscillators with time-delayed coupling

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We carry out theoretical and experimental studies of $m : n$ synchronization of two pulse oscillators with time-delayed coupling. In the theoretical study we use the concept of phase resetting curves and analyze the system dynamics in the case of weak coupling. We derive a Poincaré map and obtain the synchronization zones in the parameter space for arbitrary rotation numbers $m : n$. To verify the theoretical results we design an electronic mimic and study its dynamics experimentally. We show that the theoretical predictions agree with the experimental observations, including location of the synchronization zones and bifurcations inside them.

1D analytical solutions to the magnetostatic problem in relativistic plasma with arbitrary energy distribution of particles. Method of invariants of particle motion

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Recent progress in analytic understanding of the origin and various properties of self-consistent quasi-static configurations of magnetic field and current structures emerging in an anisotropic collisionless multicomponent plasma with arbitrary energy distribution of particles is reviewed. It the original part of the talk, following a method of invariants of particle motion applied to the planar current sheets and cylindrically symmetric filaments [1,2], we find analytically a wide class of self-consistent magnetostatic configurations in a collisionless plasma admitting essentially arbitrary energy distribution of particles. The theory is based on Grad-Shafranov type equation and automatically takes into account complicated motion of both trapped and non-trapped particles, as well as spatial inhomogeneity of their anisotropic distribution functions. We describe general properties and typical magnetic field profiles of all qualitatively different one-dimensional multicomponent current structures in a polynomial-exponential type of particle distribution functions, including relativistic case.

We consider collisionless stationary neutral plasma and assume translational symmetry along z axis. We assume that the current density is oriented along the same z axis and we describe the magnetic field, which lies in $x-y$ plane, by vector potential $\mathbf{A}(x,y) = A(x,y)\mathbf{z}_0$.

Then, for each particle the total momentum p and generalized momentum P_z along the z axis are conserved. If for each particle species j we take the particle distribution function (PDF) to be a function of these two integrals of motion

$$F_j = F_j(p, p_z + e_j A(x,y)/c), \tag{5}$$

then it satisfies the Vlasov equation. From Maxwell equations only one non-trivial equation remains

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\frac{4\pi}{c}\sum_j e_j \int \hat{F}_j(p, p_z + e_j A/c)\frac{p_z}{m_j\gamma_j} d^3\mathbf{p}, \tag{6}$$

where e_j, m_j, and $\gamma_j = (1 + p^2/m_j^2c^2)^{1/2}$ are the charge, mass, and Lorentz factor of the particles of species j.

Integration over p_z in (6) can be carried out analytically for a wide class of distributions

$$\hat{F}_j = \sum_l \exp\left(\zeta_{jl}\frac{p_z + e_j A/c}{m_j c}\right)\sum_{i=0}^{d} \hat{F}_{jli}(p)\left(\frac{p_z + e_j A/c}{m_j c}\right)^i. \tag{7}$$

The result is a Grad-Shafranov equation

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\frac{dU}{dA}, \tag{8}$$

where we introduce the so-called Grad-Shafranov potential

$$U = 8\pi^2 \sum_j m_j^2 c^3 \sum_l \exp(\zeta_{jl} a_j) \sum_{m=0}^{d} a_j^m C_{jlm}, \tag{9}$$

$$C_{jlm} = \sum_{i=m}^{d} \int \hat{F}_{jli}(p)\left[Y_{jlmi}\left(\frac{-\zeta_{jl}p}{m_j c}\right) - Y_{jlmi}\left(\frac{\zeta_{jl}p}{m_j c}\right)\right] p\frac{dp}{\gamma_j},$$

$$Y_{jlmi}(b) = \frac{n!\exp(-b)(-\zeta_{jl})^{m-i-3}}{2\ (i-m)!m!} \cdot \big(\exp(b)\Gamma(i-m+1,b)\left[(i-m+2)(i-m+1)\right.$$

$$\left. +b^{i-m+2} + (i-m+2)b^{i-m+1}\big)\right.,$$

$a_j = e_j A/m_j c^2$ is dimensionless vector potential.

Classification of the current sheets is the following.

If magnetic field changes sign more than once, then the solution is periodic. The Grad-Shafranov potential $U(A)$ has the form of potential well and $A(x)$ describes (in general, nonlinear) oscillations in this well. The profiles of parts of $B(x)$ of different sign are mirror images of each other. Corresponding density of current is symmetric w.r.t. to every plane of zero magnetic field.

Non-periodic solutions arise when $A(x \to \pm\infty)$ either correspond to local maxima of $U(A)$ or go to infinity. This may be the same maximum (or the same infinity), in which case $A(x)$ bounces off a potential wall in $U(A)$ and therefore $B(x)$ is antisymmetric with exactly one sign reversal at the turning point. Corresponding current density is symmetric, but can change its sign any even number of times.

Alternatively, $A(x \to \pm\infty)$ may correspond to two different maxima of $U(A)$ of the same height (or go to infinities of different sign and could complicate analytical evaluation of the Grad-Shafranov potential $U(A)$), in which case magnetic field has the same sign everywhere. Corresponding current density must change its sign at least once, and net current is zero.

If $A(x \to \infty)$ corresponds to a finite maximum of $U(A)$, this generally means that $B(x \to \infty)$ decreases exponentially (together with current density).

If $A(x \to \infty) \to \infty$ magnetic field far away from the sheet cannot be exponentially small, and generally either demonstrates power-law decay (not faster than $1/x$) or approaches a constant value (as a rule, monotonically). In the first case current density decreases not faster than x^{-2} (possibly oscillating with sign alternations, in contrast to fixed sign of magnetic field), net current is zero. It the second case the current may be localized with arbitrary power-law index or even exponentially, and the net current is nonzero.

We do not consider the case where $B(x \to +\infty) \neq B(x \to -\infty)$, since this kind of magnetic field would require external currents "at infinity" to support it.

Similar classification is possible for cylindrically symmetric solutions (current filaments).

In the final part of the talk, several typical examples of self-consistent current filaments and sheets with arbitrary energy PDFs are described. On this basis we investigate various properties of current filaments and sheets, including magnetic energy content, gyroradius to thickness ratio, PDF anisotropy, and synchrotron radiation. We conclude that the approach presented opens the possibility to analyical modelling of current structures observed in cosmic and laboratory plasmas as well as obtained in numerical simulations.

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Sequential activity in neuronal ensembles with excitatory couplings

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Many dynamic processes in neuronal ensembles can be viewed as sequential switching activity between elements or groups of elements. This sequential neuronal activity could be explained by various physiological functions of the nervous system. It seems extremely important to study sequential activity in neural networks with regard to nonlinear dynamics. There is a hypothesis that sequential activity in neural networks can be accounted for by the existence of a stable heteroclinic circuit in the phase space of a dynamical system that models activity in the neural network. The basic principle underlying sequential switching activity generation is winnerless competition (WLC) principle. The main idea of WLC is that there exists a stable heteroclinic circuit between singular trajectory of the saddle type in the phase space (i.e. saddle equilibrium states, saddle limit cycles etc.) and representative point moves in vicinity of this heteroclinic circuit. If a representative point is moving in the vicinity of a certain saddle trajectory, then a certain neuron or group of neurons is activating. Thus, a stable heteroclinic circuit in the phase space is a mathematical representation of sequential switching activity in ensembles of coupled neurons. A necessary condition for such behavior is the presence of inhibitory couplings between neurons. In this study we investigate the problem of switching activity in the ensemble of excitatory coupled elements. In this study we proposed a new model of neural ensembles. The elements were connected with each other by excitatory couplings. We studied models both with mutual and unidirectional couplings. It was found that if an asymmetric coupling takes place, then there appears a stable heteroclinic cycle in the phase space of the system. A stable heteroclinic cycle also exists in the ensemble of inhibitorily coupled neurons. However, the ensemble with mutual couplings does not have a stable heteroclinic cycle but has a stable limit cycle. In addition, we studied the perturbed system. It was discovered that this system also has a stable limit cycle instead of a stable heteroclinic one.

Multistability of oscillatory regimes in chains of rotators

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We show that different regimes can coexist in systems of locally diffusively coupled rotators elements for the same parameters values. For example, in-phase synchronization and regime of global synchronization caused by the soliton-like wave propagation are observed depending on initial conditions only. We classify synchronous regimes into “fast” and “slow” and give an analytical estimation for the synchronization frequency of the “fast” regime. We also provide an estimation on the number of “slow” regimes coexisting for the same parameters and its dependence on the chain length.

Chaotic dynamics for some models of systems with dry friction

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We consider a simple dynamical system that includes a body, moving under action of an external force and a dry fricton element. Such system is the simplest model for the percussion drilling process. First of all, using properties of the considered friction element, we reduce such system to a discontinuous map of the segment. Controlling points of discontinuity, we are able to apply classical theory of 1D maps and, consequently, find out chaotic and periodic regimes.

The obtained type of chaos seems to have never been theoretically obtained for single degree of freedom mechanical systems. However, its existence is confirmed by numerical simulations and experiments (Wiercigroch, Krivtsov, Ing, Pavlovskaia et al).

Our approach could be applied to some systems with non-uniqueness of solutions that can be caused by impacts and other strongly nonlinear phenomena.

Transient dynamics in ensemble of inhibitory coupled Rulkov maps

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Nowadays there are multiple experimental evidences that sequential switchings of activity between individual neurons and groups of neurons governs processes in different neuronal systems[1-3]. This activity also underlies cognitive processes [4]. In the simple case ensembles that are able to demonstrate predescribed types of activity contain only a few neurons.

We study three Rulkov maps [5-7] with mutual inhibitory couplings. In order to receive more biological relevant description of couplings we consider main features of real biolological inhibitory couplings, such as dependence of postsynaptic element activity level on presynaptic element activity level and inertia of couplings. Constructed in such a way model is discrete and so it is very easy to numerical analysis.

We study numerically different dynamical regimes that can be obtained in this motif by governing coupling parameters. In particular, we focus on two main regimes in such systems: sequential activity regime and multistable regime, and also study bifurcation transition from one regime to another.

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О классификации диффеоморфизмов 3-многообразий с одномерными просторно расположенными базисными множествами.

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В докладе рассматриваются А-диффеоморфизмы (то есть диффеоморфизмы, удовлетворяющие аксиоме А. С. Смейла), заданные на замкнутом связном 3-многообразии M^3. Согласно спектральной теореме С. Смейла неблуждающее множество $NW(f)$ любого А-диффеоморфизма f представляется в виде конечного объединения попарно непересекающихся замкнутых инвариантных *базисных множеств*, каждое из которых содержит всюду плотную траекторию.

В случае если $f : M^3 \to M^3$ принадлежит классу G, состоящему из А-диффеоморфизмов неблуждающее множество которых состоит только из поверхностных (принадлежащих замкнутой инвариантной поверхности) двумерных базисных множеств, наблюдается тесная взаимосвязь между топологией многообразия M^3 и динамикой рассматриваемого диффеоморфизма. А именно, в [1], [2],[3] доказано, что многообразие M^3 в этом случае является локально тривиальным расслоением над окружностью со слоем тор. В работах [3], [4] выделен класс Φ модельных диффеоморфизмов на многообразии $M_{\widehat{J}} = \mathbb{T}^2 \times \mathbb{R}/_\sim$, где $(z, r) \sim (\widehat{J}(z), r-1)$ для некоторого алгебраического автоморфизма $\widehat{J}$ тора $\mathbb{T}^2$, заданного матрицей $J \in GL(2, \mathbb{Z})$, которая либо является гиперболической, либо $J = \pm Id$. Каждый диффеоморфизм $\phi \in \Phi$ локально является прямым произведением алгебраического автоморфизма тора $\mathbb{T}^2$, заданного гиперболической матрицей $C \in GL(2, \mathbb{Z})$, такой что $CJ = JC$, и структурно устойчивого диффеоморфизма окружности $\mathbb{S}^1$. Доказано, что каждый диффеоморфизм $f \in G$ является Ω-сопряженным некоторому диффеоморфизму $\phi \in \Phi$, и более того, если f является структурно устойчивым, то он топологически сопряжен с ϕ.

В докладе рассматривается класс G_1 A-диффеоморфизмов, заданных на трехмерных многообразиях и таких что, неблуждающее множество любого диффеоморфизма из G_1 принадлежит объединению конечного числа двумерных поверхностей, каждая из которых является вложением двумерного тора и содержит одномерное канонически вложенное и просторно расположенное базисное множество. При естественных ограничениях на структуру пересечения инвариантных двумерных многообразий точек из таких базисных множеств, устанавливается полусопряженность любого диффеоморфизма из G_1 модельному диффеоморфизму из класса Φ.

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Deformations of functions on surfaces by symplectic difeomorphisms

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Let M be a closed orientable surface. Then the group of diffeomorphisms $D(M)$ of M naturally acts from the right on the space $C^\infty(M)$ of smooth functions on M. In particular, for each $f \in C^\infty(M)$ one can define the corresponding stabilizer

$$S(f) = \{f \circ h = f \mid h \in D(M)\}$$

and the orbit

$$O(f) = \{f \circ h \mid h \in D(M)\}.$$

In a recent series of papers the author described the homotopy types of connected components of $S(f)$ and $O(f)$ for a large class of smooth functions on M which includes all Morse functions. The aim of this talk is to show that many of these results extends to the action of the group $Symp(M, \omega)$ of symplectic diffeomorphisms of M with respect to any symplectic form (area) ω on M.

Evolving dynamical networks with transient cluster activity

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We study transient sequential dynamics of evolving dynamical networks, i.e., those having active nodes and links and activity-dependent topology.

We show that such networks can generate sequences of metastable cluster states where each state is a cyclic sequence of clusters following each other in a certain order. We found the way how the sequences generated by such networks can be robust against background noise, small perturbations of initial conditions, and parameter detuning, and at the same time, can be sensitive to input information.

Homoclinic orbits in certain multidimensional maps

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In this talk we consider a certain map with one scalar nonlinearity.

For the unimodal continuous functions having a bounded away from zero discontinuous derivative we prove the existence of a parameter domain for which the map has a singularly hyperbolic attractor. In the case of unimodal smooth functions we consider the limiting sets of the map, which can be studied in terms of symbolic dynamics.

We prove the existence of a bifurcational set leading to the emergence of different homoclinic orbits.

On Limit Cycles, Resonances and Chaos in Nearly Integrable Hamiltonian Systems

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The role of limit cycles in autonomous two-dimensional quasi-Hamiltonian systems in the case of periodical perturbations is discussed. Several examples illustrate the following phenomena:

1. Transition of a limit cycle through a resonance zone;
2. Formation of a limit cycle and its bifurcations in a resonance zone;
3. Formation of quasi-attractors as a result of destruction of stable limit cycles.

Our work was partially supported by the Russian Scientific Foundation, grant 14-41-00044 and the Ministry of education and science of Russian Federation, project 1410.

Generation of bursting activity in coupled neurons with post-inhibitory rebound

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The control of rhythmic activity (swimming, running, etc) is provided by the small neuronal subsystems, that are called Central Pattern Generators (CPGs). CPGs consist of a small number of neuron cells with different types of coupling and demonstrate different patterns of bursting activity. It's also believed that post-inhibitory rebound plays an important role in CPGs functioning. In our work we consider dynamic of half-center oscillator (HCO), the two inhibitory coupled slightly different neurons with rebound, that can be considered as a building block of CPGs. We examine the linkage of post-inhibitory rebound mechanism and generation of bursting activity in HCO for different cases of endogenous dynamics.

Nonlinear stochastic dynamics of sensory hair cells

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Sensory hair cells are mechanoreceptors transducing mechanical stimuli to electrical signals in auditory and vestibular periphery in vertebrates. In amphibians, hair cells exhibit spontaneous activity in their hair bundles and membrane potentials, reflecting two distinct active amplification mechanisms employed in these peripheral mechanosensors. We use a two-compartment model of bullfrog's saccular hair cell to study how the interaction between its mechanical and electrical compartments affects the emergence of distinct dynamical regimes, and the role of this interaction in shaping the response of the hair cell to weak mechanical stimuli. The model employs a Hodgkin-Huxley type system for the basolateral electrical compartment and a nonlinear stochastic hair bundle oscillator for the mechanical compartment, which are coupled bidirectionally. Consistent with experiments, the model demonstrates that dynamical regimes of the hair bundle change in response to variations in the conductances of basolateral ion channels. We show that sensitivity of the hair cell to weak mechanical stimuli can be maximized by varying coupling strength, and that stochasticity of the hair bundle compartment is a limiting factor of the sensitivity.

On adiabatic invariance for billiards in magnetic field

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Billiard in a magnetic field is a popular model in nonlinear dynamics. In this model the motion of a charged particle in a plane region with a perfectly reflecting smooth boundary is considered. A magnetic field is perpendicular to the plane of particle motion. If the magnetic field is strong enough, then a drift of a particle along billiard's boundary is possible. For this drift, a particle performs a skipping motion along the boundary with numerous collisions with it. Segments of particle's trajectory between collisions are close to an arc of Larmor circle. The radius of the Larmor circle and the distance of its centre from the boundary change slowly in the course of the drift due to non-uniformness of the magnetic field. As a result, the Larmor circle may recede from the boundary. Then the particle drifts in a non-uniform magnetic field for a long time without collisions with the boundary. This gradient drift is described by the classical guiding centre theory. If the trajectory of the gradient drift ends at a new collision with the boundary, then the process is repeated. We show that the drift along the boundary has an adiabatic invariant. Change of regime of motion from the drift along the boundary to the gradient drift without collisions with the boundary leads to a quasi-random jump of adiabatic invariant. We obtain an asymptotic formula for this jump. We demonstrate that an accumulation of results of these jumps for multiple changes of regimes of motion leads to destruction of adiabatic invariance in a large domain of the phase space.

Evolving dynamical networks with transient cluster activity

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We study transient sequential dynamics of evolving dynamical networks, i.e., those having active nodes and links and activity-dependent topology. We show that such networks can generate sequences of metastable cluster states where each state is a cyclic sequence of clusters following each other in a certain order. We found the way how the sequences generated by such networks can be robust against background noise, small perturbations of initial conditions, and parameter detuning, and at the same time, can be sensitive to input information.

Transition to chaos in 2(n+1)-dimensional system consisting of n dynamically coupled nonlinear oscillators

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In this work we study local and nonlocal bifurcations that change the dynamics of $2(n+1)$-dimensional system consisting of n identical nonlinear oscillators with coupling defined by a linear second-order differential equation. Linear stability analysis shows that the system has 3^n saddle-type equilibria. The manifolds of these states can form a network of heteroclinic linkages, and, as a result, complicated dynamics with coexisting regular and chaotic attractors can be observed. To examine attractors' structures and bifurcations leading to their emergence, we first consider the case when n identical subsystems are in synchronous mode. In this case the dynamics of the system occurs on 4-dimensional manifold where the system has only three steady states: the type of the trivial steady state depends on the system parameters, two nontrivial equilibria are always of saddle type. We specify the parameter range where Shilnikov conditions for these nontrivial states are satisfied and prove the existence of heteroclinic contour that connects these saddle-foci in symmetrical case (or homoclinic orbit connecting one of these saddle-foci to itself for asymmetrical case). Within this range

coexistence of wide variety of different regular and chaotic attractors is observed [1]. Moreover, we reveal a range where the divergence of the vector field on the leading 3-dimensional manifolds of the saddle-foci is positive, the system has homoclinic orbit or heteroclinic contour and therefore chaotic attracting sets become wild. The detailed analysis shows that the basins of these attractors are relatively small and have riddled structure. Finally, we demonstrate that in $2(n+1)$-dimensional phase space of the system these attractors are persistent under transversal perturbations.

This work is supported by the Russian Foundation for Basic Research (projects 12-01-00694 and 14-02-31727).

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Interaction-based transition from passivity to excitability

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In this work we suggest a mechanism of construction of excitable medium from passive dynamical elements. The idea is to couple the passive systems with the excitable ones. In the case of cardiology one may think of a site of fibrosis tissue in heart. Under normal conditions this piece of tissue can not conduct electrical pulses and thus serves as an obstacle which may cause arrhythmia to arise. We showed that a piece of excitable tissue attached above the passive area of cardiac muscle can turn the problematic region into excitable medium able to propagate signals. Again, in the case of cardiology this could be possibly done via surgery. We demonstrated that the result of the interaction of excitable and passive cells would be the transformation of the latter into the excitable units. This could happen for a really large parameter (coupling) ranges. Moreover, the new tissue would be stable with respect to small perturbations according to the restitution curve studies. The speed and with of the propagated pulse, however, would differ from those in the originally excitable medium. This would result in the in-homogeneity of the excitation pattern. The consequences of such effect are to be studied further.

We also demonstrated the generality of the effect with the FHN system, gave the analytic qualitative description of the process in terms of phase space analysis and obtained the dependency of the excitation threshold on coupling strength that align nicely with numerical simulation results.

Energy function for rough 3-diffeomorphisms with two-dimensional non-wandering set

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We consider the structurally stable diffeomorphisms defined on a closed orientable 3-manifold. Assuming that the non-wandering set of the diffeomorphism has topological dimension two, we prove that it has an energy function.

Потапов В.И.

В работе проведено сравнение механизмов рождения хаотических аттракторов в однопараметрической системе Франческини-Тибальди[1] и трёхпараметрической системе Рикитаки[2]. Установлено, что система Франческини-Тибальди в пятимерном пространстве имеет семь состояний равновесия и при прохождении критического значения числа Рейнольдса r=22.85370163183116527 в окрестностях седло-фокусов(3,2) рождаются изолированные предельные циклы, то есть имеет место локальная бифуркация Андронова-Хопфа. Далее, при увеличении параметра r до 28.8, в R5 образуется сложное притягивающее множество фазовых траекторий: происходит глобальная бифуркация, перестройка фазовой картины в области, содержащей все четыре седло-фокуса[3].

В пятимерном пространстве образуется многообразие с фрактальной размерностью 4<D<5 -это хаотический аттрактор Франческини-Тибальди. Такая «водная» модель может объяснять происхождение турбулентных процессов в течение жидкости. Аналогично динамический хаос воспроизводит и четырёхмерная система Рикитаки, но механизм его другой. В этой математической модели [4] двухдискового динамо Рикитаки, адекватно описывающей инверсии магнитной оси земного шара, имеется три состояния равновесия: два симметричных устойчивых фокуса-узла и одно седло (3,1). При значениях резистивной диссипации ?=0,2 и коэффициентов ?1=0,022 и ?2=0,02 вязкого трения происходит рождение перекрученного цикла в R4 из сепаратрис седла(3,1). Далее, при ?=1 ?1=0,008 ?2=0,002 топология состояний равновесия не меняется, перехода корней через мнимую ось нет, но в четырехмерном пространстве формируется сложное притягивающее множество фазовых траекторий с хаотическими осцилляциями как токов, так и угловых скоростей дисков. Хотя в этой системе локальной бифуркации Андронова-Хопфа и нет, однако возможно при малых шевелениях параметров разрушение сепаратрисного цикла и образование многообразия с фрактальной размерностью 3<D<4.Обе рассмотренные системы являются диссипативными во всём фазовом пространстве.

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Improvement of diagnostic methods of microwave radiometry.

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The task of monitoring of the condition of a damaged the area body of a patient without removing the protective bandage is very important in the treatment of complex burns. IR cameras are not able to see a surface of the patient's body even through one a layer of dry bandages. Our examination of patients with burns using the methods of passive-active radiometry in 8 - mm range [1] showed the possibility of solving this problem. Measurements have shown, that operation of the radiometer in passive or passive-active mode, the resulting distribution of self-radiation or reflection coefficient in the burn area [2] and under the bandage, and without a bandage similar. However there remain a number of unresolved until the end of the tasks.

The report compares the main characteristics and results of infrared and microwave radiometry. In the report objects and phenomena are presented that require further investigation in order to improve quality by microwave radiometry. They include distortion maps diagnosis, which occurs as a result of a decrease in resolution (of the order of the wavelength of the received radiation) and the curvature of surface the patient's body. Power of its own microwave radiation is much less than in the IR range, which requires increasing the accumulation time of the signal. Since the reflection coefficient of the human body for microwaves is much greater than that for the IR waves, it is necessary to consider the radiation of local objects around the patient. This radiation is reflected from the patient's body and creates on of diagnostic map a false heterogeneity. Quickly identify this heterogeneity does not allow a significant amount of time for create the one radio image.

The report suggests ways to improve the equipment, methods, algorithms of obtaining and processing information. As a result, in contrast to existing bulky stationary installations, doctors are will receive a mobile device. The doctor will be able quickly to change the speed and trajectory of the scan that will allow him to quickly find local heterogeneity. After that, the doctor will be able carefully and deeply investigate this heterogeneity. According to his desire, the doctor will be able to change the tilt axis, focus and rotate the antenna in relation to the diagnosed surfaces. In parallel, the computer accumulates the measured signals, creates the radio image of the investigated area and simultaneously combines it with his video.

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Generalized Rikitake systems and their applications

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Application of the Rikitake model namely two-disk dynamo system for investigation of geomagnetic field is well known [1]. Up to now this model is considered as purely mathematical model. However it is obvious that if one makes physical model of this system and chooses its parameters properly then one would obtain chaotic output voltage that is one would obtain generator of chaos. The device described do not include semiconducting elements therefore it will be stable under the action of extremal conditions of exploitation such as radiation or high temperature. This property of generator of chaos is urgent [2].

In the report generalizations of the Rikitake system possessing by 2n-dimensional phase space on systems from n connected dynamo are considered. One way of generalization of these systems is an investigation of both regular and irregular lattices on plane from interacting magnetic dynamo. The simplest example of such system is the system from three connected dynamo. Another way of generalization of the Rikitake model is a research of linear and ring structures consisting of identical weakly connected two-disk dynamo.

At small n one can carry out numerical investigation of these systems by means of program WInSet [3], but at large n one has to use parallel variants of Runge-Kutta method and supercomputers. Perspectives of experimental research of these systems on physical models interpreting as n interacting magnetic vortexes in kernel of Earth by means of oscilloscopes and frequency response analysers are also discussed.

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Model description of the dynamic air platform trajectory instability within the bounds of Hamiltonian formalism.

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When shaping radar images by means of airborne radar with synthetic aperture antenna, the principal problem is air platform trajectory instabilities. These air platform trajectory instabilities generate modulating interference, which can cause complete degradation of spatial resolution of the radar images. In the paper [1] vector of air platform trajectory instability is found for the coaxial-rotor helicopter, regarded as a carrier for the airborne radar. The most important feature of this design of aircraft, which considerably improves stability and handling characteristics, is aerodynamic symmetry. Due to aerodynamic symmetry there is practically no connection between longitudinal and lateral motion on the coaxial-rotor helicopter, so we can change a real helicopter by a model system, namely spherical pendulum. In truth we always have small deviations of a mapping line from the straight line. In the report it is supposed that there are no axial rotations of the pendulum. And vector of these deviations is thought to rotate uniformly along the small circle. In this case one can reduce problem investigated to the well-known problem about motion of an electron in field of two different longitudinal electromagnetic waves in collisionless plasma [2]. Unperturbed Hamiltonian function of this system proves to be Hamiltonian function of simple pendulum. Thus theory of perturbations is constructed easily in action-angle variables [2]. Therefore the consideration of the air platform trajectory instability and the consideration of the function of interference modulation one can carry out with arbitrary degree of accuracy. The results obtained will be used for the further improvement of range resolution of the synthetic aperture radar.

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Intrinsic Strong Shape in Dynamical Systems

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The interaction between topology and dynamical systems will be considered through shape theory tools. Namely, using the intrinsic approach to shape, the notion of a proximate sequence will be introduced as a sequence of near continuous functions which converge in homotopical sense and a natural way of producing one in a given flow will be discussed. The central part of the presentation will be the proof of the strong shape theorem for global attractors in compact metric spaces using the intrinsic approach to shape which combines continuity up to a covering and the corresponding homotopies of second order.

Noise-induced bursting generation in Hindmarsh-Rose model

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Hindmarsh-Rose model [1] was developed to describe bursting, one of the most important types of neural activity. It is a mode when intervals of periodic spiking alternate with intervals of resting.

We study the effects of random disturbances on the Hindmarsh-Rose model. Due to the strong nonlinearity, even the original deterministic system demonstrates very diverse complex dynamic regimes. Random perturbations considerably affect the mechanisms of excitation in neuronal systems. Even small stochastic fluctuations can lead to a significant qualitative changes in the nonlinear dynamics of such systems.

We consider a parametrical zone where the deterministic system has the only stable equilibrium. We show that under random disturbances, a stochastic generation of high amplitude oscillations occurs. The system demonstrates an alternation of small fluctuations near the equilibrium with high amplitude oscillations, that can be called noise-induced bursting generation.

For the analysis of this phenomenon we propose a method based on the stochastic sensitivity function technique [2].

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Falling Motion of a Circular Cylinder Interacting Dynamically with N Point Vortices

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The problem of falling motion of a body in fluid has a long history and was considered in a series of the classical and modern papers. Some of the effects described in the papers, such as periodic rotation (tumbling), can be encountered only in viscous fluids and thus demand for their proper treatment the use of the Navier - Stokes equations with boundary conditions specified on the body's surface. As a rule, such problems are hardly amenable to analytical analysis and can be addressed only numerically.

Another approach is to use (instead of the exact Navier - Stokes equations) some phenomenological ODE models which capture the viscous effects qualitatively.

In this paper we study the influence of the vorticity on the falling body in a trivial setting: a body (circular cylinder) subject to gravity is interacting dynamically with N point vortices. The circulation around the cylinder is not necessarily zero. So the model we consider here is exact and, at the same time, not so despairingly complex as most of the existing models are.

The dynamical behavior of a heavy circular cylinder and N point vortices in an unbounded volume of ideal liquid is considered. The liquid is assumed to be irrotational and at rest at infinity. The circulation about the cylinder is different from zero. The governing equations are presented in Hamiltonian form. Integrals of motion are found. Allowable types of trajectories are discussed in the case of single vortex. The stability of finding equilibrium solutions is investigated and some remarkable types of partial solutions of the system are presented. Poincare sections of the system demonstrate chaotic behavior of dynamics, which indicates a non-integrability of the system.

Simultaneous influence of AMPA and NMDA receptor currents on a neuron model with differential responses

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Several types of neurons (dopaminergic, noradrenergic, serotonin-containing neurons, etc.) demonstrate response differentiation, i.e. they respond qualitatively different to different excitatory stimuli. In particular, the NMDA receptor (NMDAR) current can significantly increase their firing frequency (more than 5-fold, compared to the tonic activity frequency) whereas AMPA receptor (AMPAR) current is not able to evoke high frequency activity and usually suppresses firing. However, both currents are produced by glutamate receptors and, consequently, in most cases are activated simultaneously. Here we take a neuron model that responds differentially to AMPA and NMDA synaptic currents and consider their simultaneous influence. Different types of neuron activity (rest state, low frequency or high frequency firing) are observed depending on the

conductance of the AMPAR and NMDAR currents. We show that the frequency increases more effectively if both receptors are activated simultaneously (for the certain parameters values) than if they are activated separately. In particular, the maximal frequency is 20% greater than that with NMDAR alone. Thus, we confirm the major role of NMDAR in the evocation of high-frequency firing and conclude that AMPAR activation further significantly increases the frequency. The dynamical mechanism of such frequency growth is explained in the framework of phase space evolution.

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Morse-Smale systems with three fixed points

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We consider Morse-Smale systems with the non-wandering set consisting of three fixed points (two nodes and a saddle). We study the topological structure of supporting manifolds and the question of classification. The results obtained in collaboration with V.S. Medvedev.

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Dynamics of gradient-like diffeomorphisms on 2-manifolds with one-dimensional invariant sets

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In 1973 M. Peixoto [2] classified the Morse-Smale flows without closed trajectories using distinguished graph.

In 1985-1987 V.Z. Grines and A.N. Bezdenezhnykh obtained topological classification of gradient-like cascades on orientable surfaces (for a detailed description of the results found in the book [1]).

We consider the class G of gradient-like diffeomorphisms on a manifold M^2. Nonwandering set of a diffeomorphism $f \in G$ is represented as $\Omega(f) = \Omega_0(f) \cup \Omega_1(f) \cup \Omega_2(f)$, where $\Omega_0(f)$, $\Omega_1(f)$ and $\Omega_2(f)$ are the sets of sink, saddle and source periodic points of diffeomorphism f respectively.

In this paper, a class $\tilde{G}$ of gradient-like diffeomorphisms of two-dimensional manifolds such that for every $f \in G$ the set $\Omega_1(f)$ is a union of $\Omega_1^+(f) \cup \Omega_1^-(f)$ such that:

1. $cl(\bigcup\limits_{\sigma\in\Omega_1^+} W^u_\sigma) \cup cl(\bigcup\limits_{\sigma\in\Omega_1^-} W^s_\sigma)$ consists of a finite number of disjoint components, each of which is homeomorphic to a circle; 2. $(\Omega_0 \cup \Omega_2) \subset (cl(\bigcup\limits_{\sigma\in\Omega_1^+} W^s_\sigma) \cup cl(\bigcup\limits_{\sigma\in\Omega_1^-} W^u_\sigma))$.

Interrelation between dynamics of such diffeomorphism and topology of the ambient manifold is studied. It was found that the ambient manifold M^2 diffeomorphism class $\tilde{G}$ is either a torus or a Klein bottle.

Under the additional restriction imposed on the class $\tilde{G}$, the problem of topological classification reduces to the classification of structurally stable diffeomorphisms of the circle, which was obtained by A. Maier in [3].

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