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"Transparent points" in Discrete NLS Equation with Saturation

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We consider standing lattice solitons for discrete nonlinear Schrödinger equation with saturation (NLSS), where so-called transparent points were recently discovered. These transparent points are the values of the governing parameter (e.g., the lattice spacing) for which the Peierls-Nabarro barrier vanishes. In order to explain the existence of transparent points, we study a solitary wave solution in the continuous NLSS and analyse the singularities of its analytic continuation in the complex plane. The existence of a quadruplet of logarithmic singularities nearest to the real axis is proven and applied to two settings: (i) the fourth-order differential equation arising as the next-order continuum approximation of the discrete NLSS and (ii) the advance-delay version of the discrete NLSS. In the context of (i), the fourth-order differential equation generally does not have solitary wave solutions. Nevertheless, we show that solitary waves solutions exist for specific values of governing parameter that form an infinite sequence. We present an asymptotic formula for the distance between two subsequent elements of the sequence in terms of the small parameter of lattice spacing. It is in excellent agreement with our numerical data. In the context of (ii), we also derive an asymptotic formula for values of lattice spacing for which approximate standing lattice solitons can be constructed. The asymptotic formula is in excellent agreement with the numerical approximations of transparent points. However, we show that the asymptotic formulas for the cases (i) and (ii) are essentially different and that the transparent points do not generally imply existence of continuous standing lattice solitons in the advance-delay version of the discrete NLSS.

Heteroclinic cycles and chaos in system of four identical phase oscillators with biharmonic coupling

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Coupled phase oscillators are known examples of systems that exhibit a variety of dynamic behaviours: synchrony, chaos, chimera states and mixed dynamics. When oscillators and interactions between them are identical, the coupling function becomes the main source of complexity. Due to results by Watanabe and Strogatz, Kuramoto-Sakaguchi coupling allows only simple behaviour, while using a biharmonic coupling function,

which includes only first two harmonics, opens up possibilities for chaos when number of oscillators is greater than 5. However, previously there were no numerical evidences that such coupling leads to chaos for systems of four phase oscillators. In this talk we discuss an approach for searching chaotic attractors close to heteroclinic cycles of this system. We also discuss few scenarios that lead to birth of chaos. This study was supported by Ministry of Science and Higher Education of Russian Federation, contract 0729-2020-0036.

Making stable the shear modulus instability

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It is commonly believed that simple cubic lattices are mechanically unstable due to softening of the shear modulus. Similarly, two-dimensional square lattices are commonly unstable and the typical triangular lattice is formed instead. We provide counterexamples and perform a simple stability analysis in terms of the sign of the second derivative of the interaction potential and the lattice spacing. As an example, we consider a two-dimensional lattice of Rydberg atoms and find specific parameters at which the usual triangular lattice is unstable. As a result, a variety of unusual lattice packings is observed in classical Monte Carlo simulations. As an additional check for the stability we perform normal mode analysis. The proposed mechanism for instability might be relevant to other systems and potentially lead to very unusual effects.

Stable burstings in a STSP model with glia's impact

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In this talk, we propose new phenomenological model of short term synaptic plasticity (STSP) with neuron-glial interactions. We found that the glia's impact leads to emerging of a stable bursting activity, which is important for the understanding of complex dynamics of neuronal networks.

Our model [1] is based on Tsodyks and Markram model of STSP and describes deterministic behavior of a population of identical excitatory neurons using three state variables: the average excitatory network activity $E(t)$ (in hertz) at any given time, and two other variables $X(t)$ and $u(t)$ that model synaptic depression and facilitation, respectively. An additional equation for variable $Y(t)$ describes the concentration of the gliatransmitter released due to a cascade of biochemical reactions in the process of neuron-glial interactions. The resulting 4-D system is written in the form [1]:

$$\begin{array}{l} \dot{E} = -E + \alpha \ln(1 + e^{\frac{JuXE+I_0}{\alpha}}), \\ \dot{X} = \frac{1-X}{\tau_D} - uXE, \\ \dot{u} = \frac{U(Y)-u}{\tau_F} + U(Y)(1-u)E, \\ \dot{Y} = \frac{-Y}{\tau_Y} + \beta H_Y(X). \end{array}$$

Here I_0 is a global inhibition. Parameter α is the threshold of the gain function and u is the release probability of the available neurotransmitter fraction, X. Parameter τ_D represents the recovery time from the depressed state. The probability of neurotransmitters release is given by $UE(t)$, with U representing the baseline level of facilitation variable $u(t)$ with τ_F as the time constant modeling facilitation. The characteristic relaxation time, τ_Y, is equal to 1 sec, $H_Y(X)$ is sigmoidal function of the form

$$H_Y(X) = \frac{1}{1 + e^{-20(X - X_{thr})}}, \tag{1}$$

where X_{thr} is the threshold activation value. In our model, the change in the release probability in the presence of a gliatransmitter is described as follows:

$$U(Y) = U_0 + \frac{\Delta U_0}{1 + e^{-50(Y - Y_{thr})}}, \tag{2}$$

where U_0 is the probability of glutamate release in the absence of astrocytic influence, the second term changes the probability of release due to astrocyte, and Y_{thr} is the threshold activation value.

The constructed model can demonstrate a rich set of temporal patterns from a quiescence (corresponding to a stable equilibrium) and tonic activity (corresponding to a stable limit cycle of period 1) to modes of regular and irregular bursting activity (corresponding to stable limit cycles of a long period and chaotic attractors).

This talk gives a description of these modes, and also considers the transition bifurcations between them. In particular, it was numerically found that an increase in I_0 leads to an increase in the number of spikes in a burst, and after a certain critical value $I_0^* \approx -1.7392$ there is a transition from bursting to spiking activity.

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[1]*Barabash N. V., Levanova T. A., Stasenko S. V.* STSP model with neuron - glial interaction produced // 2021 Third International Conference Neurotechnologies and Neurointerfaces (CNN). 2021. P. 12–15.

Two dimensional auxiliary systems method

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For the first time the 2D auxiliary (comparison) systems approach 50 years ago at the ODE conference in Sverdlovsk. At this conference, in particular, it was presented the theorem on the persistent homoclinic orbit existence in the nonautonomous equation without small parameters

$$\ddot{\varphi} + g(\varphi)\dot{\varphi} + F(\varphi) = ah(t),$$

where all three functions are periodic. Later this method was developped for the multidimensional autonomous system

$$\begin{aligned} \dot{\varphi} &= l^T x - a\Phi(\varphi), \\ \dot{x} &= Ax + b\Phi(\varphi), \end{aligned}$$

(startup of Nekorkin V.I. scientific activity), where A is $n \times n$–Hurwitz matrix and $\Phi(\varphi)$ is a periodic function. In 1983 I submited doctoral dissertation entitled "Method of 2D auxiliary systems in qualitative theory of certain systems".

This talk presents new approaches of the method and recent examples of its use. Bifurcational analysis is presented for the system of identical oscillators with inertial Huygens coupling

$$\begin{aligned} \ddot{x}_i + g(x_i)\dot{x}_i + f(x_i) &= -\mu\ddot{y}, \\ \ddot{y} + h\dot{y} + \Omega^2 y &= -\mu \sum \ddot{x}_i. \end{aligned}$$

The second order Kuramoto model

$$\beta\ddot{\varphi}_i + \dot{\varphi}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin(\varphi_j - \varphi_i), \qquad i = 1, 2, \ldots, N,$$

is considered. The conditions of partial synchronization are obtained with the help of 2D auxiliary systems.

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Non-classical homoclinic bifurcation in a piecewise-smooth Lorenz-type system

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Consider piecewise-smooth dynamical Lorenz-type system, composed of three linear systems of ODE [1] A_s, A_l, and A_r :

$$
A_s: \begin{array}{l} \dot{x} = x, \\ \dot{y} = -\alpha y, \\ \dot{z} = -\nu z, \end{array}
$$

$$
A_l: \begin{array}{l} \dot{x} = -\lambda(x+1) + \omega(z-b), \\ \dot{y} = -\delta(y+1), \\ \dot{z} = -\omega(x+1) - \lambda(z-b), \end{array} \qquad A_r: \begin{array}{l} \dot{x} = -\lambda(x-1) - \omega(z-b), \\ \dot{y} = -\delta(y-1), \\ \dot{z} = \omega(x-1) - \lambda(z-b), \end{array} \tag{1}
$$

where α, δ, ν, ω, λ and b are positive. These linear systems (subsystems) are defined in the following partition of the phase space G_s, G_l and G_r, respectively:

$$
G_s : |x| < 1, y \in R^1, z < b,
$$

$$
G_l : \begin{cases} x \le -1 \quad \text{for} \quad z \le b, \\ x \le -1 \quad \text{for} \quad z > b, y \ge 0, \\ x < 1 \quad \text{for} \quad z > b, y < 0, \end{cases} \qquad G_r : \begin{cases} x \ge 1 \quad \text{for} \quad z \le b, \\ x \ge 1 \quad \text{for} \quad z > b, y < 0, \\ x > -1 \quad \text{for} \quad z > b, y \ge 0. \end{cases}
$$

Linear subsystem A_s determines the dynamics of (1) in G_s and has saddle O_s at the origin. Subsystem $A_{r,l}$ are defined in $G_{r,l}$ and have symmetrical 3-D foci $e_{r,l} = \{\pm 1, \pm 1, b\}$, respectively. We assume

$$
\frac{1}{2} < \nu < 1.
$$

which makes saddle value $\sigma = 1 - \nu > 0$ positive. Introduce new parameters

$$
\gamma = b e^{-\frac{3\pi\lambda}{2\omega}}, \quad \gamma_{cr} = 2\sqrt{1 + \lambda^2/\omega^2} e^{-\frac{\delta}{\omega} \arctan \frac{\lambda}{\omega}},
$$
$$
\mu = (\gamma - 1)\gamma^{\frac{1}{\nu-1}}, \quad \varepsilon = (\gamma - \gamma_{cr})\gamma^{\frac{1}{\nu-1}}.
$$

Then the following theorem is true.

Theorem. (unstable homoclinic orbit generates a stable limit cycle)

1. For $\mu < \varepsilon \le 0$, system (1) has two stable foci e_l, e_r, and a saddle O_s.
2. For $\mu = 0$, $\varepsilon = 0$, two symmetrical unstable homoclinic orbits of saddle O_s (homoclinic butterfly) are formed in system (1).
3. For $\varepsilon > 0$, increasing $\mu \in (\varepsilon, \varepsilon + \varepsilon^{1/\nu})$, leads to emerging of a period 2 stable limit cycle and two saddle limit cycles in system (1), that were borned simultaneously from the homoclinic butterfly.

A proof of the theorem is considered in this talk. These results are presented in papers [2,3].

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On the collision of a chaotic attractor with a chaotic repeller in a skew three-dimensional Anosov-Möbius map

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In this work, we study a diffeomorphism given on a three-dimensional torus

$$\begin{aligned} x_{n+1} &= M(\varepsilon, 0, c + \mu \sin(2\pi t_n + \alpha)) x_n, \\ t_{n+1} &= 2t_n + s_n, \\ s_{n+1} &= t_n + s_n, \end{aligned} \tag{1}$$

Here t, s, x are phase variables and $\varepsilon, \alpha, \mu, c$ are parameters. Note that the map for variables t and s is exactly the well-known Anosov map of the two-dimensional torus and $M_{\varepsilon,u,v}(x)$ is a Möbius map

$$M_{\varepsilon,u,v}(x) = v + \frac{1}{2\pi}\arg((1+\varepsilon^2)\cos(2\pi(x-u)) + 2\varepsilon + i(1-\varepsilon^2)\sin(2\pi(x-u))).$$

At some regions of parameter values, this map has a chaotic attractor separated from a chaotic repeller. Note that all periodic orbits of the attractor have the two-dimensional stable and one-dimensional unstable manifolds while all periodic orbits of the repeller have the one-dimensional stable and two-dimensional unstable manifolds. With changes in parameter values, these attractor and repeller can collide in one chaotic "set" corresponding to mixed dynamics. We show that this collision occurs via a simple (codimension-1) tangent bifurcation when some saddle periodic orbit belonging to the attractor merges with the corresponding saddle periodic orbit belonging to the repeller. We find a boundary of the existence of separated attractor and repeller in the (α, c)-parameter plane and show that it consists of pieces of tangent bifurcation curves for different periodic orbits.

We stress that after the collision of the chaotic attractor and repeller the newly born chaotic set has a heterodimensional cycles connecting pairs of periodic saddle orbits of different indices. According to Li and Turaev theory [1], the existence of such cycles leads to the robust existence of hyperchaotic orbits inside this chaotic set. Thus, the map under consideration gives an example of when a simple tangent bifurcation leads to a significant complication of chaotic dynamics.

This work is supported by the RSF grant 19-71-10048.

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Радиосвязь на хаотических сигналах в дециметровом диапазоне частот

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Прямохаотические средства связи были предложены 20 лет назад. Первые эксперименты были проведены в 2000-2001 годах и продемонстрировали работоспособность предложенного подхода. В нем были заложены две базовые идеи, которые впервые позволили реализовать использование хаотических сигналов в системах беспроводной связи: формирование хаотического сигнала непосредственно в радио- или СВЧ диапазоне, использование для передачи простейшей схемы модуляции «включил – выключил». В некотором смысле прямым следствием этих идей стало использование импульсных хаотических сигналов, а не непрерывных, и применение на приемной стороне детектора огибающей, т.е. использование некогерентного энергетического приема. Эти исходные работы послужили основой для того, чтобы испытать возможности предложенной технологии в системных проектах по разработке экспериментальных макетов приемопередающих средств беспроводной сверхширокополосной связи. Результатом этих исследований стало принятие технологии сверхширокополосной беспроводной передачи данных в качестве опционального решения в стандарт IEEE 802.15.4a (2007 г). Тем самым международное научно-техническое сообщество впервые признало хаотические сигналы в качестве перспективных носителей информации для беспроводных систем связи. В период с 2007 года по настоящее время был проведен ряд НИР и НИОКР, в результате которых было создано несколько вариантов приемопередатчиков малого радиуса для связи как в режиме «точка – точка», так и в составе беспроводных локальных сетей связи и сенсорных сетей. В качестве рабочего диапазона частот использовался диапазон 3. . . 5 ГГц. В докладе рассматривается задача оценки потенциальных возможностей средств прямохотической связи в метровом и дециметровом диапазонах длин волн, создание экспериментальных макетов сверхширокополосных приемопередатчиков в полосе частот, входящей в эти диапазоны, и их экспериментального исследования в лабораторных и полевых условиях. Этот диапазон для таких средств связи используется впервые. Предварительные оценки показывали, что здесь можно рассчитывать на значительное увеличение дальности передачи при тех же пиковых мощностях передатчиков, что и у приемопередатчиков повышенной дальности в диапазоне 3. . . 5 ГГц. В целом эти оценки оправдались. Однако при этом выявился ряд особенностей нового диапазона, которые необходимо учитывать при разработке конкретных образцов приемопередающей аппаратуры.

Periodic trajectories of geometrically integrable maps in the plane

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The coexistence problem of periods of periodic trajectories is solved for geometrically integrable maps in the plane, at first, continuous; at second, discontinuous generalized 2D-Lorenz maps with Lorenz-quotients derived from symmetric unimodal interval maps.

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Dynamics in a phase model of half-center oscillator with inhibitory couplings

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It is well known that the vital functions of animals and humans, such as heartbeat, breathing and locomotion, are regulated by central pattern generators (CPG). CPGs are circuits in self-contained integrative nervous systems, able to generate and control basic repetitive patterns of coordinated motor behavior without sensory feedback or peripheral input. Biological experiments show that the universal component of CPG is the half-center oscillator.

In this paper we will consider a new model of a half-center which is consisting of two coupled phase oscillators. Each element of the ensemble is an excited neuron. The interaction between elements implemented through inhibitory couplings. The case of identical and non-identical ensemble elements was considered. in both cases, bifurcation diagrams were obtained for the parameters responsible for the strength and duration of the inhibitory effect.

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Real chaos, and complex time

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Real vector fields $\dot{z} = F(z)$ in $\mathbb{R}^N$ extend to $\mathbb{C}^N$, for entire F. We do not impose restrictions on the dimension N. The homoclinic orbit $z(t) = 1 - 3/\cosh^2(t/\sqrt{2})$ of the pendulum $\ddot{z} = z^2 - 1$ is a planar example. Note the double poles at complex times $t/\sqrt{2} = \mathrm{i}\pi/2 + k\pi$, for integer k.

One manifestation of adiabatic elimination, infinite order averaging, or "invisible chaos" are exponentially small upper bounds

$$C(\eta)\exp(-\eta/\varepsilon)$$

on homoclinic splittings under discretizations of step size $\varepsilon > 0$, or under rapid forcings of that period. Here $\eta > 0$ should be less than the distance of any complex poles of the homoclinic orbit $z(t)$ from the real axis. However, what if $z(t)$ itself is entire, and η can be chosen arbitrarily large?

We consider connecting orbits $z(t)$ between limiting hyperbolic equilibria $v_\pm$, for real $t \to \pm\infty$. We assume separately nonresonant real unstable eigenvalues, at v_- , and stable eigenvalues, at v_+ . Locally, we study the resulting irrational torus flows, in imaginary time. Globally, we conclude the existence of singularities of $z(t)$ in complex time t. In that sense, real connecting orbits are accompanied by finite time blow-up, in imaginary time.

Multi-scale magnetic field structures in an expanding elongated plasma cloud with hot electrons subject to an external magnetic field

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We analyze the system of Maxwell-Vlasov equations and carry out the appropriate 3D and 2D particle-in-cells simulations of the expansion of a magnetized plasma that initially uniformly fills a half-space and contains a semi-cylindrical region of heated electrons elongated along the surface of the plasma boundary. This geometry is related, for instance, to the ablation of a plane target by a femtosecond laser beam under quasi-cylindrical focusing. We find that the decay of the inhomogeneous plasma–vacuum discontinuity is strongly affected by an external magnetic field parallel to its boundary. We observe various transient phenomena, including the anisotropic scattering of electrons and the accompanying Weibel instability, and reveal various spatial structures of the arising magnetic field and current, including multiple flying apart filaments of a z-pinch type and slowly evolving current sheets with different orientations. The magnitude of the self-generated magnetic field can be of the order of or significantly exceed that of the external one. Such phenomena are expected in the laser and cosmic plasmas, including the explosive processes in the planetary magnetospheres and stellar coronal arches. The open problems, related to both physics and mathematics under consideration, are discussed [1]. The laboratory-astrophysics part of the work, especially related to the 3D simulations, was supported by the Russian Science Foundation, project no. 21-12-00416. Simulations were carried out in the Joint Supercomputer Center of the Russian Academy of Sciences.

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Asynchronous chaos and bifurcations in a model of two coupled Hindmarsh-Rose neurons

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We consider a minimal possible network of two interacting neurons simulated by the Hindmarsh-Rose model with linear coupling. The three-dimensional Hindmarsh-Rose model is a convenient model of neural activity, capable of capturing all the desirable types of activation patterns of a neuron: resting, spiking and bursting (both regular and irregular) [1, 2, 3, 4]. Hence, there exist extensive studies of dynamics in the single neuron model (see, e.g. [5, 6, 2, 7] and references therein).

One can find single-neuron dynamical regimes in a model of a network of identical neurons as invariant sets embedded in the synchronization manifold, if the coupling force vanishes there. However they can be unstable. On the other hand, synchronization in neural networks is considered important in biological applications, since synchronous periodic firing of a group of neurons is supposed lead to pathological behaviour, such as epileptic seizures [8].

We use the values of parameters from [7]. We find that the synchronous regimes are unstable in a wide range of the control parameter. Among the synchronous regimes that remain stable, there are only those corresponding to resting and regular spiking patterns of neural activity, while all the solutions corresponding to synchronous bursting are unstable. On the other hand, we find a new asynchronous dynamical regimes, which are monostable in a wide range of control parameters. The activation patterns corresponding to the asynchronous dynamics change with the control parameter: one can observe transition from spiking to bursting, while both of them can be regular or irregular.

We identify that asynchronous chaotic dynamics emerge according to the Afraimovich-Shilnikov scenario. We also find domains of bistability, in which synchronous and asynchronous regimes coexist.

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Singularly embedded solitons in the fifth order KdV model

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In this talk we will discuss embedded solitons in a generalisation of the KdV equation in connection with bifurcations and the singular perturbation theory.

It is well known that embedded solitons correspond to homoclinic orbits associated with a saddle-centre equilibrium in a reversible Hamiltonian system. The model considered in this talk depends on two parameters which opens possibility for emergence of homoclinic solutions at a reversible Hamiltonian (O^2, $i\omega$) bifurcation. In particular, Tovbis and Pelinovsky (2006) showed that existence of homoclinic solutions is related to simple zeroes of a Stokes constant. We will discuss various definitions of the Stokes constant and compare some numerical methods for its evaluation. We will also see that for large values of a parameter the Stokes constant originates from a singularly perturbed equation which leads to existence of infinitely many simple zeroes.

This talk is based on a joint work-in-progress with R.Barrio, A.R.Champneys, V. Gelfreich, J.T. Lázaro, J.R.Pacha.

Beyond all orders breakdown of the homoclinic connection to L_3

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We consider the Restricted Planar Circular 3-Body Problem (RPC3BP) with ratio between the masses of the primaries μ small. This configuration can be modeled by a two degrees of freedom Hamiltonian. It has a saddle-center equilibrium point called L_3 (collinear with the primaries and beyond the largest one) with a 1-dimensional stable and unstable manifold. Moreover, the modulus of the hyperbolic eigenvalues are weaker, by a factor of order $\sqrt{\mu}$, than the elliptic ones.

In this talk, we present an asymptotic formula for the distance between the stable and unstable manifolds of L_3. Due to the rapidly rotating dynamics, this distance is exponentially small with respect to $\sqrt{\mu}$ and, as a result, classical perturbative methods (i.e the Melnikov-Poincaré method) can not be applied.

This is a joint work with I. Baldomá and M. Guardia.

Self-oscillating system with weak dissipation demonstrating the «stochastic web» in the conservative limit: approximations, evolution of attractors and chaos in the phase space

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It is known that the pulse driven van der Pol oscillator

$$\ddot{x} - (\gamma - \mu x^2)\dot{x} + \omega_0 x = \sum_{m=-\infty}^{\infty} F(x)\delta(t - nT) \tag{1}$$

which for small values of the parameters γ and μ can also be regarded as a linear oscillator with a small dissipative self-oscillatory perturbation can demonstrate various phenomena of nonlinear dynamics depending on the choice of the external forcing amplitude function $F(x)$. In various limit cases the «stochastic web» - in the conservative limit for $F(x) = \lambda \cos x$ [1] - and «Hamiltonian» critical behavior - in the dissipative case for a certain choice of the parameter values and, for example, the quadratic function $F(x)$ [2] – can be obtained. Earlier we have shown that in the phase space of the «intermediate» models obtained by choosing the function $F(x)$ as an expansion of $\cos x$ in a Taylor series, a sequence of saddle-node bifurcations appears with a decrease of the nonlinear dissipation parameter μ. The number of bifurcations increases with the complication of the form of the external forcing amplitude function $F(x)$ [3]. In order to simplify the bifurcation analysis one can turn from the ODE (1) to a discrete mapping [2]

$$x_{n+1} = B\frac{F(x_n) + y_n}{\sqrt{1 + C[x_n^2 + (F(x_n) + y_n)^2]}}, y_{n+1} = -B\frac{x_n}{\sqrt{1 + C[x_n^2 + (F(x_n) + y_n)^2]}} \qquad (2)$$

using the approximate solution of the autonomous equation in the intervals between external pulses. Bifurcations in a discrete mapping could be analyzed using specialized software, for example, *Content* [4].

In order to understand changes in the dynamics of the systems with different external forcing while dissipation approaches zero we investigate the structure of the phase portraits and bifurcation diagrams. Scenarios of the evolution of attractors with the decrease of dissipation are revealed, and the difference of these scenarios the for different types of functions of external forcing is shown.

Since the conservative model with harmonic amplitude function generates the «stochastic web» and allows the unlimited diffusion through the stochastic layer, we also investigate the phenomena of diffusion and transitional chaos in the system with weak dissipation. In order to understand how the additional dissipation affects the diffusion process we obtain basins of attraction combined with the charts of the transient time in the phase plane. We also calculate the values of Lyapunov exponents for trajectories starting from a set of initial conditions in order to find the trajectories with existence of transitional chaos and to determine its lengths.

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Relaxation solutions in a logistic equation with state-in-the-past-dependent delay

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Consider logistic equation with state-in-the-past-dependent delay:

$$\dot{N} = \lambda N \left[1 - N(t - h(\lambda) - f(N(t - L)))\right], \qquad (1)$$

where λ is sufficiently large ($\lambda \gg 1$) and $L >= 0$. Under certain assumptions about functions $h(\lambda)$ and $f(N)$, the next theorem holds:

Theorem 1. *If $\lambda \gg 1$, then original equation has nonlocal relaxation periodic solution $N^*(t, \lambda)$. The initial condition of this solution belongs to the convex, bounded and closed set.*

Cases with $L = 0$ and $L > 0$ are slightly different, and the case $L = 0$ was studied in detail in [1].

Asymptotic properties of solution $N^*(t, \lambda)$ were also investigated. Namely, if $\lambda \gg 1$, then the period and the amplitude of this solution are asymptotically large, and its minimal value is asymptotically small.

We used the method of the big parameter [2] in order to establish these facts.

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Reversible perturbations of Hénon-like maps

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For area-preserving Hénon-like maps and their compositions, we consider smooth perturbations that keep the reversibility of the initial maps but destroy their conservativity. For constructing such perturbations, we use two methods, a new method based on reversible properties of maps written in the so-called cross-form, and the classical Quispel-Roberts method based on a variation of involutions of the initial map.

We study symmetry breaking bifurcations in one-parameter families of reversible non-conservative Hénon-like maps, using the constructed perturbations. We show that the simplest bifurcations of this type are reversible pitchfork bifurcations of periodic orbits. We consider such families in the cases of the product of two Hénon maps with the Jacobians b and b^{-1}, the nonorientable conservative Hénon map and the orientable conservative Hénon map. In the first two cases, we show that even symmetric fixed points can undergo pitchfork bifurcations and recover their structure. It is interesting that, in the case of orientable conservative Hénon map, this bifurcation occurs starting only from orbits of period 6 (no such bifurcation takes place for orbits of less period), that is very surprising.

On reversible systems with Lorenz attractors and repellers

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We give a short review on discrete homoclinic attractors. Such strange attractors contain only one saddle fixed point and, hence, entirely its unstable invariant manifold. We discuss the most important peculiarities of these attractors such as their geometric and homoclinic structures, phenomenological scenarios of their appearance, pseudohyperbolic properties etc.

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Localization of attractors of maps using auxiliary one-dimensional maps

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Consider an arbitrary map $\Phi : \mathbb{R}^{m+n} \to \mathbb{R}^{m+n}$ of the form $(x, y) \to (P(x, y), Q(x, y))$, and reduced map $\Phi_0 : (x, y) \to (P(x, y), y)$, where $x \in \mathbb{R}^m, y \in \mathbb{R}^n$ and $y = const$ for Φ_0. Our problem is to derive the conditions for the map Φ as well as for the boundaries of a domain D such that: 1) $\Phi D \subset D$; 2) $D = D_x \times D_y$ (direct product).

Comparison principle [1]. Assume that there exist some compacts D_x and D_y such that:
1. $\Phi_0 D_x \subset D_x$ for any $y = const$ from compact D_y;
2. $Q(x, y) \in D_y$ for any $x \in D_x$ and $y \in D_y$.
Then D is invariant under the map.

From this principle it follows that the map has an attractor $A = \Phi A, A \subset D$.

First example is a multidimensional map with a piecewise-smooth periodic nonlinear function [2] of form F_1:

$$(x,\ y_i) \to (x + \delta - ag(x) + \sum_{i=1}^{n} y_i,\ \lambda_i(-b_i g(x) + y_i)), \quad i = \overline{1, n}, \tag{1}$$

where $x \in S^1, y \in R^n, g(x) = g(x + 2\pi), a, \delta, \lambda_i, b_i$ are parameters, $|g'(x)| > h$.

Applying the comparison principle for the map F_1 as the auxiliary map Φ_0 we consider the map F_0 in the form $(x, y) \to (x - ag(x) + Y, y)$, where $Y = \sum_{i=1}^{n} y_i$ is a parameter. This map is one parameter $y = const$ family of one dimensional maps f_1: $\overline{x} = x - ag(x) + Y$. As for each f_1 the first condition of the comparison principle is fulfilled, so the interval $[c, d] = D_x$. It is easy to verify that there exists an interval $[y^-, y^+] = D_y$, satisfying the condition 2 of the comparison principle.

These conditions provide a simple technical rule for the system: from $x \in [c, d]$ and $Y \in [y^-, y^+]$ it follows that $\overline{x} \in [c, d]$. Under the condition $F_1 D \subset D$ the next theorem holds. We prove the existence of absorbing domain D containing the attractors of F_1. Namely, the next statement is true.

Theorem 1 [2]. *Let $0 < \lambda_i < \lambda^* < 1$, $i = \overline{1, n}$. Then the solid torus $D = \{\|y\| < A, x \in S^1\}$ is the absorbing domain $D, F_1 D \subset D$ and, therefore this map has an attractor $A \subset D$.*

For the second example we consider a multidimensional Henon map of a general form F_2 [3]

$$\begin{cases} \bar{x} = f(x) + \sum\limits_{i=1}^{n} a_i v_i, \\ \bar{v_1} = x, \\ \bar{v_i} = v_{i-1}, i = \overline{2, n}, \end{cases} \tag{2}$$

where $a_i \in \mathbb{R}^1, f(x)$ is the quadratic function of the form $f(x) = \mu - x^2$, $\mu \in \mathbb{R}^1$. After some transformations, the map (2) has the form [4, 5]

$$\begin{cases} \bar{x} = x + Y + F(x); \\ \bar{y_1} = -a_1\big(Y + F(x)\big); \\ \bar{y_i} = -q_{i-1} y_{i-1} - a_i\big(Y + F(x)\big), i = \overline{2, n}, \end{cases} \tag{3}$$

where $Y = \sum\limits_{j=1}^{n} y_j$; $q_1 = \frac{a_2}{a_1}, q_2 = \frac{a_3}{a_2}, ..., q_{n-1} = \frac{a_n}{a_{n-1}}$; $A = \sum\limits_{j=1}^{n} a_j$ and the function $F(x)$ is written in the form $F(x) = \mu - (1 - A)x - x^2$.

If we set $a < \left|\frac{a_{j+1}}{a_j}\right| = q_j < q < 1$; $A^+ = \sum\limits_{j=1}^{n} |a_j|$; $A = \sum\limits_{j=1}^{n} a_j > 0$ that the map (3) has absorbing domain $G_y = \{y|\ Y < \gamma\}$ for y_i coordinates. With these constraints it is easy to show that the second condition is satisfied in the comparison principle.

Applying the comparison principle for the map F_2 as the auxiliary map Φ_0 we consider the map F_0 in the form $(x, y) \to (\widetilde{\mu} \pm Y - x^2, y)$, where $Y = \sum_{i=1}^{n} y_i, \widetilde{\mu} = \mu + \frac{A^2}{4} - \frac{A}{2}$ are parameters. This map is one parameter $y = const$ family of one dimensional maps $f_2 : \bar{x} = \widetilde{\mu} \pm Y - x^2 = g^{\pm}(x)$ in domain G_y.

As for each f_2 the first condition of the comparison principle is fulfilled, so the interval $[c, d] = D_x$, where $c = x_1^-$ is the unstable fixed point of the mapping $g^-(x)$ and $d = \widetilde{\mu} + \gamma$ is the largest image of the maps $g^-(x)$ and $g^+(x)$.

Theorem 2 [4, 5]. *Let* $a < \left|\frac{a_{j+1}}{a_j}\right| = q_j < q < 1$; $A^+ = \sum\limits_{j=1}^{n} |a_j|$; $A = \sum\limits_{j=1}^{n} a_j > 0$, $c = x_1^-$ *is the unstable fixed point of the mapping* $g^-(x)$ *and* $d = \widetilde{\mu} + \gamma$ *is the largest image of the maps* $g^-(x)$ *and* $g^+(x)$. *Then the domain* $G = \{x, y | c \leq x \leq d, Y \leq \gamma\}$ *is the absorbing domain* $G, F_2 G \subset G$ *and, therefore this map has an attractor* $A \subset G$.

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Multistability and hyperchaos in models of coupled repressilators

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In this work we present some results of the detailed two-parameter bifurcation analysis in models of coupled identical repressilators. We show that the coexistence of two nested invariant tori (previously observed in these systems [1, 2]) is explained by the occurrence of specific codimansion-two bifurcations: when these systems have a symmetric periodic orbit with a pair of multipliers (+1,+1). Also we study bifurcation mechanisms leading from stable invariant tori to chaotic and, further, hyperchaotic attractors in these systems.

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Synchronous patterns in Kuramoto networks with adaptive couplings

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The processes of synchronization and cluster formation are investigated in complex networks of phase oscillators with adaptive couplings. Several scenarios of the hierarchical formation of complex synchronous states have been found, depending on the properties of the coupling adaptation function. The process represents the sequential emergence of frequency clusters in the network, obeying a certain hierarchy of sizes of the formed groups of synchronized oscillators. We studied a network consisting of two populations of phase oscillators, the interaction of which is determined by different rules for the coupling adaptation. We found that the introduction of various adaptation rules can lead to the emergence of each population complex non-stationary behavior called transient circulant clusters. In such states, each population contains a pair of anti-phase clusters whose size and composition slowly change over time as a result of successive transitions of oscillators between clusters. We show that an increase in the mismatch of the adaptation rules makes it possible to stop the process of rearrangement of clusters in one or both populations of the network. Transitions to such modes are always preceded by the appearance of solitary states in one of the populations. In a network of pulse-coupled oscillators with adaptive coupling, we discovered a dynamical mode which we call an "itinerant chimera". Similarly as in classical chimera states, the network splits into two domains, the coherent and the incoherent. The drastic difference is that the composition of the domains is volatile, i.e., the oscillators demonstrate spontaneous switching between the domains.

Infinite Turing Bifurcations in Chains of Van der Pol Systems

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A chain of the coupled systems of the Van der Pol equations is considered. We study the local – in the vicinities of the zero equilibrium state – dynamics of this chain. We make a transition to the system with a continuous spatial variable assuming that the number of elements in the chain is large enough. The critical cases corresponding to the Turing bifurcations are identified. It is shown that they have infinite dimension. Special nonlinear parabolic equations are proposed on the basis of the asymptotic algorithm. Their nonlocal dynamics describes the local behavior of solutions to the original system. In a number of cases, normalized parabolic equations with two spatial variables arise while considering the most important diffusion type couplings. It has been established, for example, that for the considered systems with a large number of elements, the dynamics change significantly with a slight change in the number of such elements.

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On new pseudohyperbolic attractor of Lorenz type

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We introduce a new type of Lorenz-like chaotic pseudohyperbolic attractor that includes 3 saddle equilibria with their one-dimensional unstable manifolds. We provide two types of governing equations. The first is the system of three ordinary differential equations with two symmetries $(x, y) \to (-x, -y)$ and $z \to -z$:

$$\begin{aligned} \dot{x} &= -x + y, \\ \dot{y} &= -\lambda y + \left(1 - y^2 - z^2\right) x, \\ \dot{z} &= -\alpha z + 2xyz. \end{aligned} \tag{4}$$

The second system is a three-dimensional Hénon map with cubic nonlinearities and symmetry $(X, Y, Z) \to (-X, -Y, -Z)$:

$$\begin{aligned} X' &= Y, \\ Y' &= Z, \\ Z' &= AZ + BX + CY - \frac{3}{2}ZY^2 + 2Z^3. \end{aligned} \tag{5}$$

Figure 1 (a) shows a "conjoined" Lorenz-like attractor of equations (1) at $\lambda = 0.04$, $\alpha = 0.2$. It visually consists of two symmetric parts, and although the orbits from the upper or the lower part do not cross the plane $z = 0$ due to the symmetry, we consider them only the parts of the conjoined attractor. The reason is we have to define a surface surrounding the absorbing domain with attractor inside, such that all the orbits intersect it transversally. The plane $z = 0$ is invariant and can not be part of surface surrounding the absorbing domain. Moreover, some orbits on the plane $z = 0$ belong to the attractor. Among these orbits are three saddle equilibria: at the origin $(0, 0, 0)$, symmetric $\left(\sqrt{1-\lambda}, \sqrt{1-\lambda}, 0\right)$ and $\left(-\sqrt{1-\lambda}, -\sqrt{1-\lambda}, 0\right)$, and heteroclinic orbits connecting them. Both upper and lower parts look similar to classic Lorenz attractor, indeed the discovered conjoined attractor is pseudohyperbolic according to numerical results.

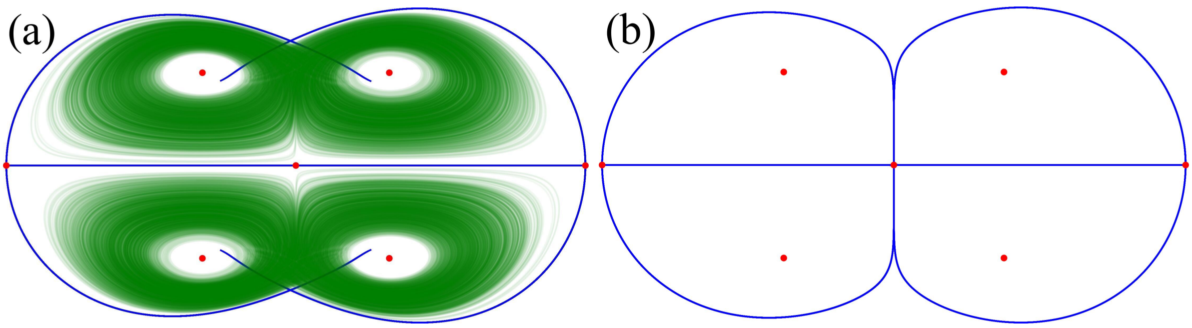

Рис. 1: orbits of (1) at $\lambda = 0.04$, $\alpha = 0.2$ (a) and at $\lambda \approx 0.29798505$, $\alpha = 0.2$ (b).

Figure 1 (b) shows the heteroclinic "butterfly" situation: unstable orbits of the symmetric saddle equilibria return to the saddle at the origin both from above and below. Similar situation for map has been hypothesized before [1]. Attractor of the map (2) is very similar to the attractor of the flow (1). The notable differences are the presence of the period-2 saddle orbits instead of two symmetric saddle points (although the period-2 orbit becomes two saddle fixed points of twice applied map) and the splitting of the separatrices connecting the fixed point at the origin with period-2 orbit. Despite the fact that the two parts of the attractor are inseparable due to the separatrix splitting, the orbits of attractor do not jump from upper to lower part or vice versa in our numerical simulations (up to 10^{12} iterations without jumps). We conjecture, that the flow system (1) is close to the normal form of the map (2) near the fixed point with triplet of multipliers $(-1, -1, 1)$.

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Invariant manifolds of two versions of the weakly dissipative complex Ginzburg-Landau equation

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We will consider the following two versions of the complex Ginzburg-Landau equation

$$u_t = u - (d + ic)u|u|^2 - (f + ih)u|u|^4 - ibu_{xx}, \tag{1}$$

$$u_t = u - (d + ic)uV(u) - (f + ih)uV^2(u) - ibu_{xx}, \tag{2}$$

where $u(t, x)$ is a complex-valued function satisfying the corresponding periodic boundary conditions

$$u(t, x + 2\pi) = u(t, x). \tag{3}$$

In our case $f, b > 0, c, d, h \in \mathbb{R}, V(u) = \frac{1}{2\pi}\int\limits_0^{2\pi} |u(t, x)|^2 dx$. The boundary value problem (1), (3) has the following single-mode periodic solution with respect to variable t

$$u_n(t, x) = \eta \exp(i\sigma_n t + inx + i\varphi_n),\ n = 0, \pm 1, \pm 2, \ldots, \tag{4}$$

where $\sigma_n = bn^2 - c\eta^2 - h\eta^4, \varphi_n \in \mathbb{R}$, and η is positive root of the corresponding quadratic equation

$$f\eta^4 + d\eta^2 - 1 = 0.$$

Theorem 1.*Let $b \in (0, b_*), b_* = 2(c + 2h)$. Then the cycles C_n, generated by solutions (4) will be local attractors if $b > b_*$ and these cycles become saddle cycles if $b \in (0, b_*)$.*

Let $b = b_* - \gamma\varepsilon, \varepsilon \in (0, \varepsilon_0)$. In this case two-dimensional invariant tori bifurcate from the cycle C_n. The post-critical (soft) and sub-critical (hard) bifurcations are possible. Asymptotic formulas are obtained for solutions that belong to $T_n(\varepsilon)$.

Analysis of the boundary value problem (2), (3) showed that it has an invariant manifold A_0 distinguished by the following condition

$$V(u) = \xi_2,$$

where ξ_2 is a positive roof of the quadratic equation $f\xi^2 + d\xi - 1 = 0$.

Let

$$A_g = \{0\} \cup A_0,$$

where $\{0\}$ is the zero equilibrium state of the studied boundary value problem (2), (3). It is shown that A_g is a global attractor for solutions of the boundary value problem (2), (3) in the sense of definition from [1,2].

Note that boundary value problems (1), (3) and (2), (3), when $f = h = 0$, were studied earlier. Such a specific case of the boundary value problem (1), (3) was studied in [3], while a similar version of the boundary value problem (2), (3) was investigated in [4,5].

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Concept of instant Lyapunov exponents and their application for numerical analysis of attractors structure

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A concept of instant Lyapunov exponents is discussed. Unlike the well known finite time ones, the instant Lyapunov exponents show expansion or contraction on infinitesimal time intervals. Two types of instant Lyapunov exponents are defined. One is related to ordinary finite time Lyapunov exponents computed in the course of standard algorithm for Lyapunov exponents. Their sums reveal instant volume expanding properties. The second type of instant Lyapunov exponents shows how covariant Lyapunov vectors grow or decay on infinitesimal time. Usage of the instant and finite time Lyapunov exponents is discussed for analysis of pseudohyperbolic attractors and saddle-node bifurcation of invariant tori. This work was supported by the Russian Science Foundation, project 21-12-00121. https://rscf.ru/en/project/21-12-00121/

Quasi-periodic bifurcations and complex dynamics in two coupled modified Anishchenko-Astakhov generators

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Radiophysical model of the modified Anishchenko-Astakhov generator is well case of nonlinear dynamical system [1-2], demonstrating multi-dimensional chaos::

$$\begin{aligned} \dot{x} &= mx + y - x\varphi - dx^3, \\ \dot{y} &= -x, \\ \dot{z} &= \varphi, \\ \dot{\varphi} &= -\gamma\varphi + \gamma\Phi(x) - gz, \end{aligned} \tag{1}$$

where $\Phi(x) = I(x)x^2$, $I(x) = \{ \begin{array}{l} 0, x > 0 \\ 1, x \leq 0. \end{array}$

Model (1) demonstrates the maximum variety of chaotic behavior [2], as well as various phenomena characteristic of four-dimensional systems, for example, the doubling bifurcation of a two-frequency torus. Within the frame of this work, we consider the features arising from the interaction of such oscillators. We considered two dissipatively coupled oscillators of type (1). In the simplest ensemble, it is possible to observe the appearance of quasiperiodic oscillations with two, three, and four incommensurate frequencies. A bifurcation of doubling of a four-frequency torus was found. The phase synchronization of quasiperiodic oscillations in a system of coupled oscillators, as well as the features of chaotic behavior resulting from the destruction of multifrequency tori are investigated.

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On the influence of boundary conditions on the dynamic properties of a logistic equation with delay and diffusion

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The important in mathematical ecology logistic equation with delay and diffusion is considered

$\frac{\partial u}{\partial t} = d\frac{\partial^2 u}{\partial x^2} - ru(x, t-1)(1+u), \quad 0 \leq x \leq 1$

and with boundary conditions

$\frac{\partial u}{\partial x}\big|_{x=0} = \kappa u|_{x=0}\,, \qquad \frac{\partial u}{\partial x}\big|_{x=1} = \gamma u|_{x=1}\,,$

where $\gamma, \kappa \in \mathbf{R}$.

It is shown that negative values of the γ parameter and positive values of κ extend the range of variation of the values of the r parameter, at which the zero equilibrium state of the boundary value problem is stable, and positive γ and negative κ – narrow.

The limiting values of the parameter r are obtained for which the zero equilibrium state is stable.

In cases close to critical in the problem of the stability of the zero solution, the analysis of the local dynamics of the boundary value problem is given. In the limiting case, for $\gamma \to -\infty$ and $\kappa \to \infty$, the analysis of the local dynamics is carried out.

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The Maxwell's billiard demon effect in simplified Tennison-Lichtenberg-Lieberman system with oscillating boundaries

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Chaotic billiards are popular models in nonlinear dynamics and chaos theory. In [1] a stadium-like billiard with oscillating boundaries was studied numerically. When the Authors studied ensemble-averaged velocity in this system, they had observed that weak oscillations of the boundary lead to the appearance of a critical value of the initial velocity in the system, below which the average velocity begins to decrease. If the initial velocity is greater than the critical velocity, then the particles are accelerated on average. This phenomenon was called billiard Maxwell's Demon.

In previous work [2], we had undertaken a research of effect billiard Maxwell's Demon in a modified model of the Tennyson-Lieberman-Lichtenberg model [3], to which bounder oscillations are added. The research had shown that the billiard Maxwell's demon is possible. The system has a limit velocity and a critical value of the initial velocity. If the initial velocity is less than the critical one, then the ensemble-averaged velocity tends to some limit. Otherwise, the average velocity increases. The scenario of the effect destruction in this system is that the limit velocity begins to increase with time, starting from a certain amplitude of the boundary oscillation.

In this work, a new system was studied, obtained by approximating expressions that describe the system from a previous work [2]. In this system, the effect of the billiard Maxwell's Demon is observed, however it is a different. There are two limit velocity in the system. The slowest particles slow down to zero velocity. Part of the particles, which have a initial velocity below the critical value, tend to the limit non-zero velocity. A new scenario of the destruction of the effect was obtained. It consists in the fact that a non-zero limit velocity tends to zero value and all trajectories begin to accelerate.

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One-dimensional factor maps of stable foliations for Lorenz and Rovella attractors

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Let X_μ be a two-parameter family of $(n+1)$-dimensional symmetric vector fields ($\mu \in \mathrm{R}^2$). Suppose that the (unperturbed) vector field X_0 has a saddle equilibrium O with a single positive eigenvalue γ, while other eigenvalues λ_i, $i = 1, \dots, n$, have negative real parts and, moreover, one of them, the nearest to 0, is real, i.e.,

(A1) $\gamma > 0 > \lambda_1 > \operatorname{Re} \lambda_i, \quad i = 2, \dots, n.$

Also, we assume that for X_0, the following holds:

(A2) one-dimensional separatrices Γ_1 and Γ_2 of the equilibrium O tend to O as $\to +\infty$ along the leading direction touching each other;

(A3) $\nu = -\frac{\lambda_1}{\gamma} = 1$;

(A4) $|A| > 1$, where A is the separatrix value of the homoclinic orbits Γ_1 and Γ_2.

L.P. Shilnikov was the first to observe (in [1]) that perturbations of the vector field X_0 may lead to chaotic dynamics. More precisely, he proved that the Lorenz attractor appears (under certain additional assumptions) when the saddle value ν becomes less than 1. Main idea used in the proof of this result is the construction of stable invariant foliation for the Poincaré map. The factorisation over this foliation reduces the problem to the study of the following one-dimensional map

$$\bar{x} = (-1 + c|x|^\nu + o(|x|^\nu)) \cdot sign(x), \tag{1}$$

Let us note that, in general, the foliation and the factor map are only $C^{1+\varepsilon}$-smooth, where ε depends on eigenvalues of X_0. This fact makes the study of map (1) more delicate, since many results from one-dimensional dynamics require greater smoothness.

In this talk, we discuss the bifurcation of the vector field when the saddle value ν becomes greater than 1; this case corresponds to the appearance of the so-called Rovella attractor [2]. We show that the reduction to one-dimensional map (1) takes place in this case as well, due to existence of the stable foliation of the Poincaré map like in the Lorenz case [3]. Based on Benedicks-Carleson technique, we prove persistence of chaotic behavior for parameter set with positive Lebesgue measure. For the vector field X_μ our main result can be stated as follows.

Theorem. *Let X_μ be a two parameter family of symmetric vector fields such that the unperturbed vector field X_0 satisfies the assumptions A1-A4. Then, in the parameter plane, there exists a set D_{RA} with positive Lebesgue measure such that $0 \in D_{RA}$ and X_μ has the Rovella attractor for all $\mu \in D_{RA}$.*

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On degenerate resonances and synchronization of quasiperiodic oscillations

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We study quasiperiodic nonconservative perturbations of two–dimensional Hamiltonian systems with nonmonotonic rotation. The behavior of solutions in a small neighborhood of a resonance level that is close to degenerate one is considered. The conditions for the existence of resonance quasiperiodic solutions are found. Special attention is paid to the bifurcations that take place in the resonance zone. As well, we study the problem of oscillations synchronization which is related to the passing of an invariant torus through the resonance zone. The results are applied to the asymmetric Duffing–Van der Pole equation.

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Universal bounds in Lorenz-like systems

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The well-known Lorenz-63 system:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= -bz + xy. \end{aligned} \tag{1}$$

describes the interaction of three modes in the problem of a convection in a two-dimensional fluid. The expression xy has a physical meaning, namely, it is the transport of heat energy, and the knowledge of the properties of this value is important for the understanding the properties of the convection.

System (1) possesses a Lyapunov function, so that all the solutions come inside a certain ball and stay there forever. This means, that for the time averages of physical values on large time intervals will be well-defined and bounded. In [1] the upper bound of the heat transport in the Lorenz system was computed. This value is achieved in the equilibrium point.

In the current work I consider system (1) with added constant offsets, these systems were introduced in [2], [3], moreover, in [2] the upper bound was computed, but it is not achieved on any solution. The results of the present work are the improvement of the heat transport upper bound obtained in [2] to a sharp value (achieved in the equilibrium) and the computation of a sharp upper bound in system from [3].

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Chaos in coupled heteroclinic cycles and its picewise-linear description

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We study two coupled identical stable heteroclinic cycles, rotating in *different* directions. In the first part, numerical simulations of the model are performed, revealing different symmetry-breaking transitions and demonstrating persistent chaos for small coupling. In the second part, an approximate approach based on a piece-wise constant representation of the dynamics is constructed. In this theory the strength of the coupling becomes irrelevant, and the only parameters are those of a single heteroclinic cycle. An one-dimensional map for the chaotic state in this representation is constructed numerically.

A heteroclinic trefoil for the Lorenz equations

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In this talk I intend to describe a recent extension to the Lorenz geometric model. The extension is defined on the entire three dimensional sphere, and for some parameters contains a heteroclinic connection that is topologically a trefoil knot. This model is strongly related to the geodesic flow on the modular surface. I'll also describe how one can prove that a similar trefoil heteroclinic connection exists for some parameters in the Lorenz equations, and how this enables us to find a relation between the geodesic flow and the actual equations at these parameters. This is joint work with Christian Bonatti.

Remark on the dynamical system in the space of double-sided sequences

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Let $x = (\ldots, x_{-n}, \ldots, x_{-1}, x_0, x_1, \ldots, x_n, \ldots)$ be the complex-valued double-sided sequence. And let L be the space of all complex-valued double-sided sequences.

If $\lambda \in L$ and $x \in L$ then one can define their product $x \star y$ as double-sided sequence with terms:

$$(\lambda \star x)_n \equiv \sum_{k=-\infty}^{+\infty} \lambda_k \, x_{n-k} \,, \tag{1}$$

thus L is commutative algebra with unit.

Further using definition (1) one can construct the following discrete dynamical system (cascade) on L:

$$x^{t+1} = \lambda \star x^t - \lambda \star x^t \star x^t \,, \qquad t \in \mathbf{Z} \,. \tag{2}$$

The report deals with system (2) in the framework of generating function $X^t(z)$ for $x^t \in L$, namely,

$$X^t(z) = \sum_{n=-\infty}^{+\infty} x_n^t z^n \,, \qquad z \in \mathbf{C} \setminus \{0\} \,. \tag{3}$$

In particular, by means of value (3) one can consider evolution in discrete time t of the following two-parametrical initial state with the next nonzero elements:

$$x_n^0 = C_{2m}^{m+n} \alpha^{m+n} \beta^{m-n} \,, \qquad |n| \leq m \,, \qquad m \in \mathbf{N} \,,$$

where C_{2m}^{m+n} are binomial coefficients.

This work continues investigations of dynamical systems on spaces of double-sided sequences started at [1].

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Periodic solutions of linear inhomogeneous differential equations with two small parameters

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The work is devoted to the construction of a T-periodic solution of a linear inhomogeneous differential equation

$$A \frac{dx}{dt} = (B_0 - \varepsilon_1 B_1 - \varepsilon_2 B_2) \, x - f(t), \tag{1}$$

here $x \in E_1$; A, B_0, B_1 и B_2 – densely defined linear Fredholm operators acting from E_1 to E_2; $f \in E_2$, $f(t+T) = f(t)$, $T > 0$; ε_1, ε_2 – small real parameters; A – degenerate or identical operator.

Here the desired T-periodic solution $x(t, \varepsilon_1, \varepsilon_2)$ to equation (1), must satisfy the condition $x(t, 0, 0) = z(t)$, where $z(t)$ — T-periodic solution of the equation [1]

$$A\frac{dz}{dt} = B_0 z - f(t). \tag{2}$$

We will solve the problem posed under the assumption that the equation

$$A\frac{dy}{dt} = B_0 y \tag{3}$$

has T-periodic solutions of the form

$$\varphi_{mj} = u_{mj} e^{i\alpha_m t}, \qquad \bar{\varphi}_{mj} = \bar{u}_{mj} e^{-i\alpha_m t}, \quad j = \overline{1, n_m},\ m = \overline{1, r}.$$

Here the numbers $\pm i\alpha_m$ ($\alpha_m = k_m \omega$, $\omega = \frac{2\pi}{T}$, $k_m \in N$, $m = \overline{1, r}$) are the A-eigenvalues of the operator B_0; u_{mj} are A-eigen elements of the operator B_0

$$B_0 u_{mj} = i\alpha_m A u_{mj}, \quad B_0 \overline{u}_{mj} = -i\alpha_m A \overline{u}_{mj},$$

$j = \overline{1, n_m}$, $m = \overline{1, r}$; n_m – geometric multiplicity of each number in a pair $\pm i\alpha_m$ (the number of A-eigen elements that correspond to an eigenvalue $i\alpha_m$).

To solve this problem, a modified Lyapunov-Schmidt [2] method is used. It is based on construction of a complete generalized Jordan collection in the sense of the works [3], [4].

It is shown in the work that equation (1), provided that the point $(\varepsilon_1, \varepsilon_2)$ belongs to a sufficiently small punctured neighborhood of zero, has a unique T–periodic analytic in ε_1 and ε_2 solution. Moreover, for $\varepsilon_1 = 0$ and $\varepsilon_2 = 0$ the solution to equation (1) goes over into the $2n$–parametric family of T–periodic solutions to equation (2).

If the side lengths of all generalized Jordan grids are equal to one, then equation (1) has a unique T–periodic solution that is analytic in ε_1 and ε_2 from a sufficiently small neighborhood of zero. Moreover, the solution of the perturbed equation (1) tends to the solution of the unperturbed equation (2) as ε_1 and ε_2 tend to zero.

If there is at least one generalized Jordan grid, the side length of which is greater than one, then the solution to equation (1) has a pole either in ε_1, or in ε_2, or simultaneously in ε_1 and ε_2.

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Integrability criteria for a family of cubic oscillators

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In this talk we consider a family of autonomous nonlinear oscillators with cubic nonlinearity with respect to the first derivative. We study equivalence problems for this family of differential equations and its integrable subcases, which are the damped harmonic oscillator and two Pailevé–Gambier type equations. As equivalence transformations we use nonlocal transformations that generalize the Sundman transformations. We explicitly construct equivalence criteria in terms of the coefficients of the considered family of equations. We also discuss the existence of first integrals and integrating factors for the members of these equivalence classes.

Moreover, we demonstrate that considered nonlocal transformations preserve autonomous invariant curves admitted by the studied family of equations. We provide explicit correlations between autonomous invariant curves and the corresponding cofactors for equations that are related via these transformations. Consequently, we see that nonlocal transformations can be used for extending known classification of invariant curves to the whole equivalence class of the corresponding equation. This means that we can obtain classification of invariant curves for a non-polynomial equation, if it belongs to an equivalence class of a polynomial equation with known classification of algebraic invariant curves. Moreover, we show that an autonomous first integral of one of two non-locally related equations can be constructed in the parametric form with the help of the general solution of the other equation.

We illustrate our results by several examples of integrable nonlinear oscillators, including intgrable cases of Rayleigh–Duffing–Van der Pol and generalized Duffing-Van der Pol oscillators and several integrable Liénard equations.

Transitions between tonic spiking and bursting in the stochastic Hindmarsh-Rose neuron model with the Lukyanov-Shilnikov bifurcation

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The Hindmarsh-Rose neuron model [1] demonstrates a wide variety of dynamical regimes and bifurcations that are associated with the complexity of neural activity. In particular, it has been shown [2] that this model can exhibit a specific bifurcation of a saddle-node periodic orbit with noncentral homoclinics known as the Lukyanov-Shilnikov bifurcation. Its feature in this model is an appearance of a bistability parameter region where two limit cycles coexist, one of them describes tonic spiking oscillations, and the other corresponds to bursting activity.

We study a stochastic variant of the Hindmarsh-Rose neuron neuron model with the Lukyanov-Shilnikov bifurcation and show that in the bistability zone, noise can induce transitions between the coexisting limit cycles. In the monostability region where the only attractor of the system is a limit cycle of the tonic spiking type, random disturbances can generate bursting oscillations. Moreover, we show that these stochastic phenomena are accompanied with coherence resonances and noise-induced transitions from order to chaos. For the parametric analysis of these effects, we use an approach that combines the stochastic sensitivity function technique [3], Mahalanobis metrics, and the method of confidence domains.

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Partially Hyperbolic Symplectic Automorphisms of 4-Torus

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We study topological properties of automorphisms of 4-dimensional torus generated by an integer matrices being symplectic either with respect to the standard symplectic structure in $\mathbf{R}^4$ or a nonstandard symplectic structure generated by an integer non-degenerate skew-symmetric unimodular matrix. Such symplectic matrix generates a partially hyperbolic automorphism of the torus, if its eigenvalues consist of a pair of real numbers outside the unit circle and a pair of complex conjugate numbers on the unit circle. The main classifying tool is the structure of a foliation generated by unstable (stable) leaves of the automorphism and the automorphism action on the center manifold.

We study the one-dimensional foliations on the torus generated by the projection on the torus of unstable and stable eigen-lines. We prove that the related foliation on the torus T^4 can be either transitive or decomposable into 2-tori. Each case requires a special investigation for its classification. Automorphisms realizing all possible cases are provided (details and proofs see in [1]).

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Escape times across a golden Cantorus and the evolution of approximant islands

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We shall consider the Chirikov standard map for values of the parameter k larger than but close to Greene's constant k_G. After a brief review of the classical Greene-MacKay renormalisation theory, we will use the corresponding scalings to investigate the phase space geometry and the dynamics near the golden Cantorus. As a collateral consequence, the computations carried out led to an estimate of k_G with 21 digits, improving the results available in the literature.

Our goal will be to study escape rates across the golden Cantorus. Accordingly, we shall pay special attention to the local dynamics around elliptic (with $|\text{trace}| < 2$) and/or reflection-hyperbolic (with trace < -2) periodic orbits whose rotation number is an approximant of the golden mean, and to the area of the stability domain that surrounds them, if any. In the references [1, 2] the behaviour of the mean of the number of iterates $\langle N_k \rangle$ to cross the Cantorus when $k \to k_G$ was described. In particular, it was shown that there exists a constant $B < 0$ such that $\langle N_k \rangle (k - k_G)^B$ becomes 1-periodic in a suitable logarithmic scale. The numerical explorations we shall present here give evidence of the shape of this periodic function and of the relation between the escape rates and the evolution of the stability islands close to the Cantorus. We will provide some final comments about the probability law of the escape rates and its behaviour as k tends to k_G.

This is a joint work with N. Miguel and C. Simó, see [3].

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